European Economic Review 111 (2019) 380–399

Contents lists available at ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/euroecorev

Procurement auctions under quality manipulation corruption

Yangguang Huang[a,*], Jijun Xia[b]

[a] Department of Economics, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China
[b] School of Economics, Shanghai University of Finance and Economics, 111 Wuchuan Rd, Shanghai 200433, China

ARTICLE INFO

Article history:
Received 19 January 2018
Accepted 5 November 2018
Available online 16 November 2018

JEL classification:
D44
D73
H57

Keywords:
Scoring auction
Quality manipulation
Corruption
Procurement

ABSTRACT

In procurement, quality manipulation corruption arises when the agent tasked with quality evaluation exaggerates the quality of a corrupt firm. If an inefficient firm is favored by the agent, the buyer can adjust the procurement mechanism such that the corruption rent of the inefficient firm erodes the technological rent of the efficient firm; however, doing so may require procuring the project at an undesirable quality level. To resolve this trade-off between corruption deterrence and quality distortion, unlike standard results in the literature, the buyer may overstate her preference for quality, and the dominance of scoring auctions over minimum-quality auctions disappears.

1. Introduction

In most procurement, the projects (objects) can be delivered at variable quality levels measured by multiple non-monetary attributes; therefore, it is indispensable to define and assess quality. However, the buyer usually does not usually possess the specific industrial expertise necessary for quality evaluation. Che (2006) notes that “In many procurement settings, ... the quality of the job provided is not easy to verify or is simply unobservable to the buyer.” Therefore, a procurement auction usually involves an *agent* (he) who intermediates between the buyer (she) and the supplying firm(s) (it/they).[1] Because the agent has some discretionary power in quality evaluation, he may use this power to seek a bribe from a *corrupt firm*. The problem of *quality manipulation* arises when the agent distorts reports of quality scores of bids. In particular, the agent can exaggerate the corrupt firm’s quality score and make it more likely to win the contract.

Kuhn and Sherman (2014) report that member states of the European Union lose approximately billion to corruption in procurement each year. They note that “The cost of corruption in public contracting is not only measured by money lost. Corruption distorts competition, can *reduce the quality*, sustainability, and safety of public projects and purchases, and reduce the likelihood that the goods and services purchased really meet the public’ s needs.” In numerous studies of corruption

* Corresponding author.
E-mail addresses: huangyg@ust.hk (Y. Huang), jijunxia@mail.shufe.edu.cn (J. Xia).

[1] In reality, the procurement task is monitored by a street-level bureaucrat, contract supervisor, or inspector. The task is then delegated to a procurement agency that specializes in that industry. The auction is usually conducted in some government-supervised trading center. A committee of experts scores the quality of received bids. Thus, there are several layers of agency between the buyer and the firm. We abstract from reality by modeling all these intermediate parties as one agent.

https://doi.org/10.1016/j.euroecorev.2018.11.001
0014-2921/

in procurement, economists have devoted particular attention to bidding rings,[2] which suppress competition and entail monetary losses to the buyer, but quality manipulation has received relatively less attention. In this paper, we study the collusion between the agent and a corrupt firm and emphasize how such corruption may distort the quality of a project and affect the buyer's payoff. Our main contribution is to show how the buyer can adjust the procurement mechanism to combat quality manipulation corruption.

In most procurement auctions, firms essentially submit bids as price-quality combinations that include items in the proposal, incumbent status, capacity, and reputation. The three most commonly used formats of multi-attribute auctions are (i) scoring auctions (request for proposals), (ii) buyer-determined auctions (request for information, design-build auctions (Takahashi, 2018), or beauty contests (Yoganarasimhan, 2015), and (iii) minimum-quality auctions (request for quotes or price-based auctions). In a scoring auction, the buyer announces and commits to a pre-announced scoring rule, and the contract is awarded to the firm that receives the highest score. In a buyer-determined auction, the buyer does not commit to a certain scoring rule and instead chooses a firm according to her preferences. If firms know the buyer's preferences, a buyer-determined auction becomes a scoring auction with the true preferences as the scoring rule.[3] In a minimum-quality auction, the buyer specifies a minimum quality standard and all bids satisfying the requirement are evaluated on price alone.[4]

Che (1993) analyzes several formats of scoring auctions and characterizes the optimal scoring rule. Under the optimal scoring rule, the buyer understates (shades) her true preference for quality based on the trade-off between information rent extraction and quality distortion. Asker and Cantillon (2008) show that scoring auctions dominate minimum-quality auctions with respect to the buyer's payoff.[5] Chen-Ritzo et al. (2005) provide experimental evidence indicating that scoring auctions dominate traditional price-only auctions. Intuitively, given the winner's quality level, rival firms are stronger in a scoring auction than in a minimum-quality auction because the former auction format allows them to propose their preferred quality levels. As a result, the winner's information rent is lower in scoring auctions and the buyer is better off.

Regarding quality manipulation, the analysis of Burguet and Che (2004) is the most relevant work to our study. In their model, an efficient firm and an inefficient firm compete for a contract in a scoring auction. Prior to the price competition in the auction, these two firms compete on bribing the agent, and the winner's quality score is exaggerated by an amount m, measuring the agent's manipulation power. A small m does not affect the allocational efficiency. However, when m is sufficiently large, although the efficient firm has advantages in both the price competition and bribery competition, it cannot guarantee that it will win the contract because the inefficient firm may spend all of its resources on either paying a high bribe or submitting a low bid. In this case, the equilibrium involves mixed strategies, and the efficient firm wins with a probability lower than one, which entails an efficiency loss.

We consider a model that differs from Burguet and Che (2004) in two respects. First, the corruption relationship is exogenously determined in our model, instead of endogenously formed through a bribery competition.[6] In reality, explicit bribery competition is nearly impossible under a well-functioning legal system, and corruption relationships are usually based on favoritism formed before procurement. Second, in addition to scoring auctions, we also study minimum-quality auctions as an important alternative auction format. Our analysis not only identifies firms' equilibrium bidding strategies but also discusses how the buyer should design the scoring rule and select auction formats under quality manipulation.

This setting allows us to separate the *technological rent* (of the efficient firm) and the *corruption rent* (of the corrupt firm), which leads to three main results of the paper. The efficient firm has a cost advantage and thus receives the technological rent.[7] Suppose that the inefficient firm is favored, its quality is exaggerated, and the resulting cost-saving is the corruption rent. This corruption rent erodes the technological rent of the efficient firm, which is in the buyer's interest. This gives rise to our **Result 1**: The buyer may be better off than in a corruption-free environment because she (sometimes) only needs to pay the difference between these two rents. In other words, the presence of corruption increases competition, and the buyer reaps the benefit. This result is the opposite of the conventional wisdom that the presence of corruption harms the buyer.[8]

[2] A bidding ring refers to a group of bidders that coordinate their bids to avoid competition within the ring. The literature on bidding rings is rich in both theoretical analysis (e.g., Graham et al., 1990; McAfee and McMillan, 1992; and Hendricks et al., 2008) and empirical studies (e.g., Porter and Zona, 1993; Pesendorfer, 2000; Bajari and Ye, 2003, and Asker, 2010).

[3] Che (1993) shows that the buyer can only use the truthful scoring rule if she lacks commitment power. Takahashi (2018) study buyer-determined auctions with uncertain evaluation.

[4] A minimum-quality auction is also subject to quality manipulation because an expert agent is needed to verify the fulfillment of the minimum quality standard.

[5] Asker and Cantillon (2008) also show that scoring auctions dominate buyer-determined and menu auctions. In a menu auction, bidders are allowed to submit multiple price-quality combination bids instead of the single bid allowed in scoring auctions. The buyer will then determine the winner and the item on its menu.

[6] Celentani and Ganuza (2002) and Huang (2018) also assume that the corruption relationship is exogenous. In the literature on bidding rings, it is common to assume that the collusion relationship is exogenously given. See, e.g., Porter and Zona (1993), Porter and Zona (1999), Bajari and Ye (2003), and Athey et al. (2011).

[7] Because the buyer does not know the efficiency (type) of the firms, the most efficient firm receives a rent due to incomplete information. The literature typically refers to this rent as the information rent. In this paper, because there is also incomplete information on the corruption relationship, we distinguish between the two rents using the terms technological rent and corruption rent.

[8] In most cases, the buyer (uninformed party) is harmed by corruption in the literature on bidding rings (e.g., Graham et al., 1990; McAfee and McMillan, 1992; and Hendricks et al., 2008), auctioneer-bidder cheating (e.g., Compte et al., 2005; Burguet and Perry, 2009; and Burguet and Perry, 2014), and quality

Note that the buyer can influence the sizes of these two rents by adjusting the scoring rule. Under standard assumptions regarding the cost function, the efficient firm's technological rent is increasing in quality. In a second-best contract, the buyer understates her preference for quality in the scoring rule to pay less of this rent. Under quality manipulation, however, if the quality is manipulated excessively, rent extraction by the efficient firm is less of an issue for the buyer than paying for the non-delivered quality from the inefficient-and-corrupt firm. In this case, the buyer may *deter corruption* by inducing a high quality level such that the efficient firm can prevail.[9] This is our **Result 2**: In selecting the scoring rule, the buyer may *overstate* her preference for quality to deter corruption.

Nevertheless, over-emphasizing quality comes at the cost of buying the project at an unnecessarily high quality. The buyer faces the trade-off between deterring corruption to prevent quality manipulation and paying for upwardly distorted quality. One way to alleviate this trade-off is to restrict firms from proposing high quality. A minimum-quality auction can serve this purpose, as it reduces firms' flexibility in quality choice. This leads to our **Result 3**: The buyer obtains a higher payoff in a minimum-quality auction because the quality is less distorted than in a scoring auction. This result resolves a paradox reported in the literature: Minimum-quality auctions are sub-optimal in theory but widely used in real-world practice (Engelbrecht-Wiggans et al., 2007) and sometimes perform better (Tran, 2011).

Note that the key driving force of these three results is that the buyer can utilize the corruption rent to erode the technological rent when these two rents belong to two firms. If the efficient firm is favored, the buyer would have to pay both rents and be worse off under quality manipulation. These results have important policy implications that will be presented in the concluding remarks.

1.1. Relationship to the literature

This paper offers new insights into two strands of literature: procurement design and corruption. Scoring auctions are commonly used in procurement with variable quality. Che (1993) finds the optimal direct revelation mechanism and shows how a scoring rule implements it. However, when the environment becomes complicated, seeking the optimal mechanism becomes difficult. Branco (1997) considers a case in which firms' costs are correlated and shows that the optimal mechanism cannot be implemented by first- or second-score auctions. Asker and Cantillon (2008) derive the equilibrium of a scoring auction with multi-dimensional private information and quality attributes, but in a subsequent contribution (Asker and Cantillon, 2010), they only find the optimal mechanism in a specific environment in which firm types are two binary distributed random variables. David et al. (2006) characterize the optimal scoring rule within the class of weighted scoring rules and demonstrate numerically that it is close to the optimal mechanism. Nishimura (2015) shows that implementing the optimal mechanism requires substantial cost complementarity among quality attributes, meaning that a weighted scoring rule cannot be optimal. Because we have incomplete information on both cost and the corruption relationship, we do not take the direct revelation mechanism approach; instead, we restrict our study to the two most popular auction formats and find the optimal way to apply them. In this way, we can characterize key factors of the procurement design problem when quality is subject to manipulation.

Delegation is inevitable in procurement: The buyer needs an agent to either oversee the entire procurement procedure or at least conduct the quality assessment. Such delegation creates numerous opportunities for corruption, such as using a sub-optimal procurement mechanism (Laffont and Tirole, 1991), allowing bid rigging (Compte et al., 2005), misrepresenting the buyer's preferences (Gretschko and Wambach, 2016), and promoting collusion among firms (Fugger et al., 2015). Quality manipulation, the focus of this paper, is a major form of corruption (Lengwiler and Wolfstetter, 2006).

In the presence of quality manipulation, procurement design becomes more complicated. In a single-firm context, Burguet (2017) finds that the optimal contract depends on whether bribes are fixed or vary with the extent of quality manipulation. When there is competition, Celentani and Ganuza (2002) show that, unlike usual auctions, increasing the competitiveness of the environment (having more firms) may not reduce corruption. We find that competition may even cause corrupt firms to win more often. Compte et al. (2005) find that, when there are quality concerns, policies that prevent some firms from bribing become ineffective. Burguet and Che (2004) and Albano et al. (2017) demonstrate that the buyer is better off by adopting a non-anonymous mechanism that handicaps some firms. We do not consider discriminating mechanisms in our analysis and instead focus only on simple and widely used auction formats. We emphasize that the scope of quality manipulation and the probability of the efficient firm being favored play important roles in the design of the procurement mechanism.

2. The model

The buyer seeks procurement of a project with variable quality $q \in \mathbb{R}_+$ among firms. If the project is delivered at quality q and the compensation is $p \in \mathbb{R}_+$, the buyer's payoff is $U(p, q) = q - p$. The buyer chooses one of the two auction formats: a scoring auction with a linear scoring rule (L) or a minimum-quality auction (M). If she chooses L, she specifies a scoring

manipulation (e.g., Celentani and Ganuza, 2002 and Burguet, 2017). Burguet and Che (2004) reach a similar result that "little manipulation power ... simply makes the efficient firm compete aggressively. Thus surprisingly, corruption benefits the buyer."

[9] In this case, the agent will not be able to help the inefficient firm secure the contract. As a result, the inefficient firm does not have an incentive to pay a bribe.

rule $S(p, q) = \alpha q - p$ by selecting a quality weight $\alpha \geq 0$, which represents the monetary equivalent of quality (Dini et al., 2006).[10] The project is awarded to the firm with the highest score. In a scoring auction, the buyer is not bound to use her true preferences as the scoring rule, instead, she can understate ($\alpha < 1$) or overstate ($\alpha > 1$) her preference for quality.[11] If the buyer chooses M, she specifies a minimum quality standard $\underline{q}$. The contract is awarded to the firm that can satisfy the quality standard at the lowest price.

After the auction format is announced, each firm submits a sealed bid in the form of a price-quality combination. Firm i is characterized by the cost function $C(q, \theta_i)$ with one-dimensionalcost parameter θ_i. If a firm wins the contract with bid (p, q), its payoff is $\pi(p, q; \theta) = p - C(q, \theta)$. A firm's payoff is normalized to zero if it does not win the contract. The cost function, $C(q, \theta)$, is strictly increasing in q and θ and twice continuously differentiable with respect to both parameters. Following Che (1993) and Burguet and Che (2004), assume that $C(0, \theta) = 0$, $C_q > 0$, $C_{qq} > 0$, $C_{q\theta} > 0$, $C_{qq\theta} > 0$, $\lim_{q\to\infty} C_q = \infty$, and $\lim_{q\to 0} C_q = 0$. We impose that $C_{qqq} > -C_{qq}^2$ as a sufficient condition for the uniqueness of equilibrium in the case without corruption (Proposition 2).

Because the buyer does not possess the necessary expertise to evaluate the quality of received bids, she hires an agent. Following Burguet and Che (2004), we assume that the agent can manipulate the evaluation by exaggerating the corrupt firm's quality score by $m \geq 0$. We refer to this parameter m as *the scope of quality manipulation*.[12] When $m = 0$, there is no quality manipulation and we say that the agent is honest. The corruption relationship is formed exogenously.[13]

The timeline of the procurement auction game is as follows: The buyer chooses the auction format and announces it. Then, firms simultaneously submit their sealed bids (p, q). Thereafter, the agent evaluates the quality score of the corrupt firm. In a scoring auction, the corrupt firm's quality score is exaggerated by m, i.e., its score is exaggerated from $S(p, q)$ to $S(p, q + m)$. The contract is awarded to the firm with the highest score. In a minimum-quality auction, the corrupt firm submits a quality $\underline{q} - m$ instead of $\underline{q}$ under quality manipulation. The contract is awarded to the auction winner. To simply the analysis, we adopt the tie-breaking rule of Burguet and Che (2004): The efficient firm wins the contract in the event of a tie.

In Sections 2.1 and 2.2, we consider a benchmark model that closely resembles the environment of Burguet and Che (2004). There are two firms, $i = 1, 2$, with type θ_1 and θ_2, respectively. Without loss of generality, assume that $\theta_1 < \theta_2$, so firm 1 is efficient and firm 2 is inefficient. We begin the analysis with the case in which the inefficient firm is always the corrupt one. Therefore, the buyer knows that there is an efficient firm ($i = 1$) and an inefficient-and-corrupt firm ($i = 2$) but does not know which one is efficient.[14] We assume complete information among firms: Each firm knows its opponent's cost parameter and corruption relationship.[15] In this benchmark model, the technological rent and the corruption rent always belong to two different firms, so the main intuition can be clearly presented. We then generalize the model to consider a positive probability of the efficient firm being favored and the case with multiple firms. In Section 3, we discuss the case with incomplete information on the corruption relationship and incomplete information on costs.

2.1. Scoring auction with linear scoring rule (L)

Suppose that the buyer adopts a linear scoring rule with quality weight $\alpha \geq 0$. As we know from Che (1993) and Burguet and Che (2004), for any given α, it is a weakly dominant strategy for firm i to choose quality $q_i(\alpha) \equiv \arg\max_q \{\alpha q - C(q, \theta_i)\}$, which maximizes the difference between the quality score and the cost of delivering this quality.[16] Note that for $\alpha = 0$, $q_1(\alpha) = q_2(\alpha) = 0$.

The maximum score[17] that the efficient firm can obtain is

$$\bar{s}_1(\alpha) = \alpha q_1(\alpha) - C(q_1(\alpha), \theta_1), \tag{1}$$

[10] There is no loss of generality from considering a one-dimensional quality measure q in the buyer's payoff function as long as the cost function is convex and the scoring rule is quasilinear. The proof can be found in Lemma 1 of Huang (2018). Suppose that the cost function is $C(\mathbf{q}, \theta)$ and the scoring rule is $S(p, \mathbf{q}) = V(\mathbf{q}) - p$, where $\mathbf{q} \in \mathbb{R}_+^L$; then, one can consider that the firm is producing a quality score $v = V(\mathbf{q})$ with cost function $C(v, \theta)$. As the form of the cost function is flexible, one can re-scale the quality measure so that it enters the payoff function linearly. The linearity of the scoring rule does impose some restrictions on the optimal scoring rule design. We consider the linearity setting due to both analytical tractability and the widespread use of weighted linear scoring rules in procurement practice.

[11] In a buyer-determined auction, if firms know the buyer's true preference, then it is as if the buyer selection $\alpha = 1$.

[12] Burguet and Che (2004) argue that if there is no monitoring or the monitoring intensity is not correlated with m, the agent will always exert his full manipulation power. We assume that preventing quality manipulation is impossible or too costly because the buyer lacks industrial expertise.

[13] We treat the process of corruption relationship formation as a "black box". Burguet (2017) "opens" this black box by using several specifications of bribery models.

[14] Under this assumption, the buyer cannot adopt a non-anonymous scoring rule that explicitly handicaps some specific firm(s) as in Burguet and Che (2004).

[15] The assumption of complete information about corruption relationship is widely used in studies of corruption in auctions (e.g., Bajari and Ye, 2003; Burguet and Perry, 2009; and Athey et al., 2011). It circumvents the difficulty of having two layers of incomplete information on both cost and the corruption relationship. An alternative approach is to assume that other bidders are unaware of corruption and thus follow the corruption-free strategy (e.g., Porter and Zona, 1993; Aryal and Gabrielli, 2013; and Tian and Liu, 2008).

[16] This holds under quality manipulation. The proof can be found in Burguet and Che (2004) and Huang (2018).

[17] This term is called productive potential in Che (1993), pseudotype in Asker and Cantillon (2008), and effective cost in Hanazono et al. (2015).

while the maximum score that the inefficient-and-corrupt firm can obtained is

$$\bar{s}_2(\alpha) = \alpha q_2(\alpha) - C(q_2(\alpha), \theta_2) + \alpha m, \tag{2}$$

where $R^L_C \equiv \alpha m$ represents the *corruption rent* of the corrupt firm. Under Bertrand-type competition, the auction outcome is determined by the magnitudes of the maximum scores. In equilibrium, the firm with a higher maximum score wins the contract by sightly outbidding the other firm.

We first consider the case when there is no quality manipulation ($m = 0$). We can easily show that the efficient firm proposes a higher quality and has a higher maximum score.

Lemma 1. *For $\theta_1 < \theta_2$, $q_1(\alpha) > q_2(\alpha)$. When $m = 0$, $\bar{s}_1(\alpha) > \bar{s}_2(\alpha)$.*

In equilibrium, the efficient firm wins the contract by proposing the price p_1 such that $s_1 = \alpha q_1(\alpha) - p_1 = \bar{s}_2(\alpha) = \alpha q_2(\alpha) - C(q_2(\alpha), \theta_2)$. Therefore, $p_1 = \alpha q_1(\alpha) - \bar{s}_2(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + C(q_2(\alpha), \theta_2)$, and firm 1 obtains profit

$$p_1 - C(q_1(\alpha), \theta_1) = \alpha q_1(\alpha) - C(q_1(\alpha), \theta_1) - \alpha q_2(\alpha) + C(q_2(\alpha), \theta_2) \equiv R^L_T,$$

which is the*technological rent* of the efficient firm.

Lemma 2. *For all $\alpha > 0$, $R^L_T > 0$ and $dR^L_T/d\alpha > 0$ and $d^2R^L_T/d\alpha^2 > 0$.*

When $m = 0$, the buyer's payoff is

$$U^L_{SB}(\alpha) = q_1(\alpha) - p_1 = q_1(\alpha) - C(q_1(\alpha), \theta_1) - R^L_T.$$

Proposition 1. *U^L_{SB} is strictly concave. There is a unique second-best quality weight $\alpha_{SB} \equiv \arg\max_{\alpha\in[0,\infty)} U^L_{SB}(\alpha)$, and $\alpha_{SB} < 1$.*

Proposition 1 shows that the buyer chooses α by the classical trade-off between extracting firm 1's rent and distorting quality from the efficient level. This implies that the buyer understates her preference for quality ($\alpha_{SB} < 1$).[18]

Next, we consider the case with quality manipulation ($m > 0$). The following lemma shows that there is a threshold quality weight $\tilde{\alpha}$ that determines the auction outcome.

Lemma 3. *For any $m > 0$, there exists $\tilde{\alpha} > 0$ such that $\bar{s}_1(\alpha) < \bar{s}_2(\alpha)$ for $\alpha < \tilde{\alpha}$; $\bar{s}_1(\tilde{\alpha}) = \bar{s}_2(\tilde{\alpha})$; and $\bar{s}_1(\alpha) > \bar{s}_2(\alpha)$ for $\alpha > \tilde{\alpha}$. $\tilde{\alpha}$ increases in m.*

In other words, the efficient firm wins the contract when the quality weight is sufficiently large. The threshold quality weight $\tilde{\alpha}$ is defined by the solution of $\bar{s}_1(\alpha) = \bar{s}_2(\alpha)$, that is,

$$R^L_T = R^L_C \Leftrightarrow \alpha q_1(\alpha) - C(q_1(\alpha), \theta_1) - \alpha q_2(\alpha) + C(q_2(\alpha), \theta_2) = \alpha m. \tag{3}$$

Let $U^L_i(\alpha)$ denote the buyer's payoff, where the subscript i indicates which firm wins the contract. The auction outcome and the buyer's payoff depends on the quality weight. By the tie-breaking rule, the efficient firm wins if $\alpha \geq \tilde{\alpha}$ and $p_1 = \alpha q_1(\alpha) - \bar{s}_2(\alpha)$. The buyer's payoff is

$$\begin{aligned} U^L_1(\alpha) &= q_1(\alpha) - p_1 = q_1(\alpha) - \alpha q_1(\alpha) + \bar{s}_2(\alpha) \\ &= (1-\alpha)q_1(\alpha) + \alpha[q_2(\alpha) + m] - C(q_2(\alpha), \theta_2) \\ &= q_1(\alpha) - C(q_1(\alpha), \theta_1) - R^L_T + R^L_C. \end{aligned} \tag{4}$$

If, instead, $\alpha < \tilde{\alpha}$, firm 2 has a larger maximum score and wins the contract. In equilibrium, it proposes the price p_2 such that

$$s_2 = \alpha q_2(\alpha) + \alpha m - p_2 = \bar{s}_1(\alpha).$$

Thus, $p_2 = \alpha q_2(\alpha) + \alpha m - \bar{s}_1(\alpha)$. In this case, the buyer's payoff is

$$\begin{aligned} U^L_2(\alpha) &= q_2(\alpha) - p_2 = q_2(\alpha) - \alpha q_2(\alpha) - \alpha m + \bar{s}_1(\alpha) \\ &= (1-\alpha)q_2(\alpha) + \alpha[q_1(\alpha) - m] - C(q_1(\alpha), \theta_1) \\ &= q_2(\alpha) - C(q_2(\alpha), \theta_2) + R^L_T - R^L_C. \end{aligned}$$

From $U^L_2(\alpha)$ and $U^L_1(\alpha)$, we can clearly see that the two rents, R^L_T and R^L_C, offset each other.

In sum, the buyer's payoff is a function with a discontinuity at $\tilde{\alpha}$:

$$U^L(\alpha) = \begin{cases} U^L_2(\alpha) & \text{for } \alpha < \tilde{\alpha}, \\ U^L_1(\alpha) & \text{for } \alpha \geq \tilde{\alpha}. \end{cases} \tag{5}$$

The selection of the optimal quality weight α^* is characterized by the following proposition.

[18] When the buyer has complete information about costs, she would procure from firm 1 and choose $\alpha_{FB} = \max_\alpha\{q_1(\alpha) - C(q_1(\alpha), \theta_1)\} = 1$.

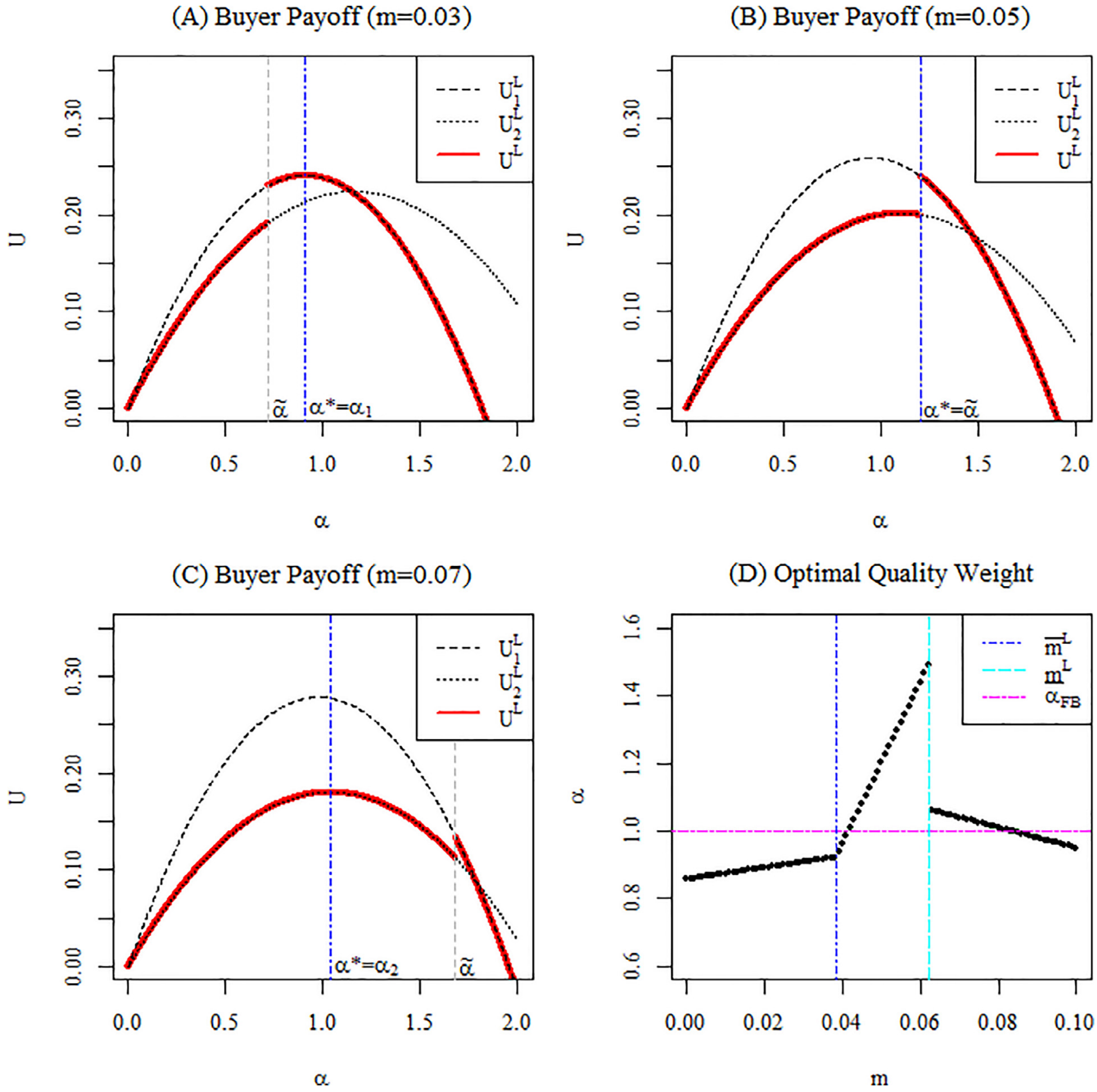

Fig. 1. Buyer payoff and optimal quality weight in L.

Proposition 2. *There exist two cutoff values of the scope of quality manipulation, $\underline{m}^L$ and $\overline{m}^L$, such that*

$$\alpha^* = \begin{cases} \alpha_1 \equiv \arg\max_{\alpha\in[0,\infty)} U_1^L(\alpha) & \text{if } m < \underline{m}^L, \\ \tilde{\alpha} & \text{if } \underline{m}^L \le m \le \overline{m}^L, \\ \alpha_2 \equiv \arg\max_{\alpha\in[0,\tilde{\alpha}]} U_2^L(\alpha) & \text{if } m > \overline{m}^L. \end{cases}$$

The optimal quality weight consists of three intervals depending on the value of m, as illustrated by Fig. 1.[19] Increasing the quality weight has two effects: First, it increases the efficient firm's technological rent relative to the corruption rent. Second, overstating quality in the scoring rule causes the buyer to pay for an unnecessarily high quality. When m is small, by selecting α_1, the technological rent dominates. In this case, the selection of quality weight mainly reflects the classical trade-off between rent extraction and quality distortion. When m becomes large, setting α_1 cannot guarantee that the efficient firm will win, so the buyer *deters corruption* by adjusting the quality weight upward to $\tilde{\alpha}$. Doing so raises the technological

[19] Figs. 1–3 are generated in the setting of Example 1 in the Appendix.

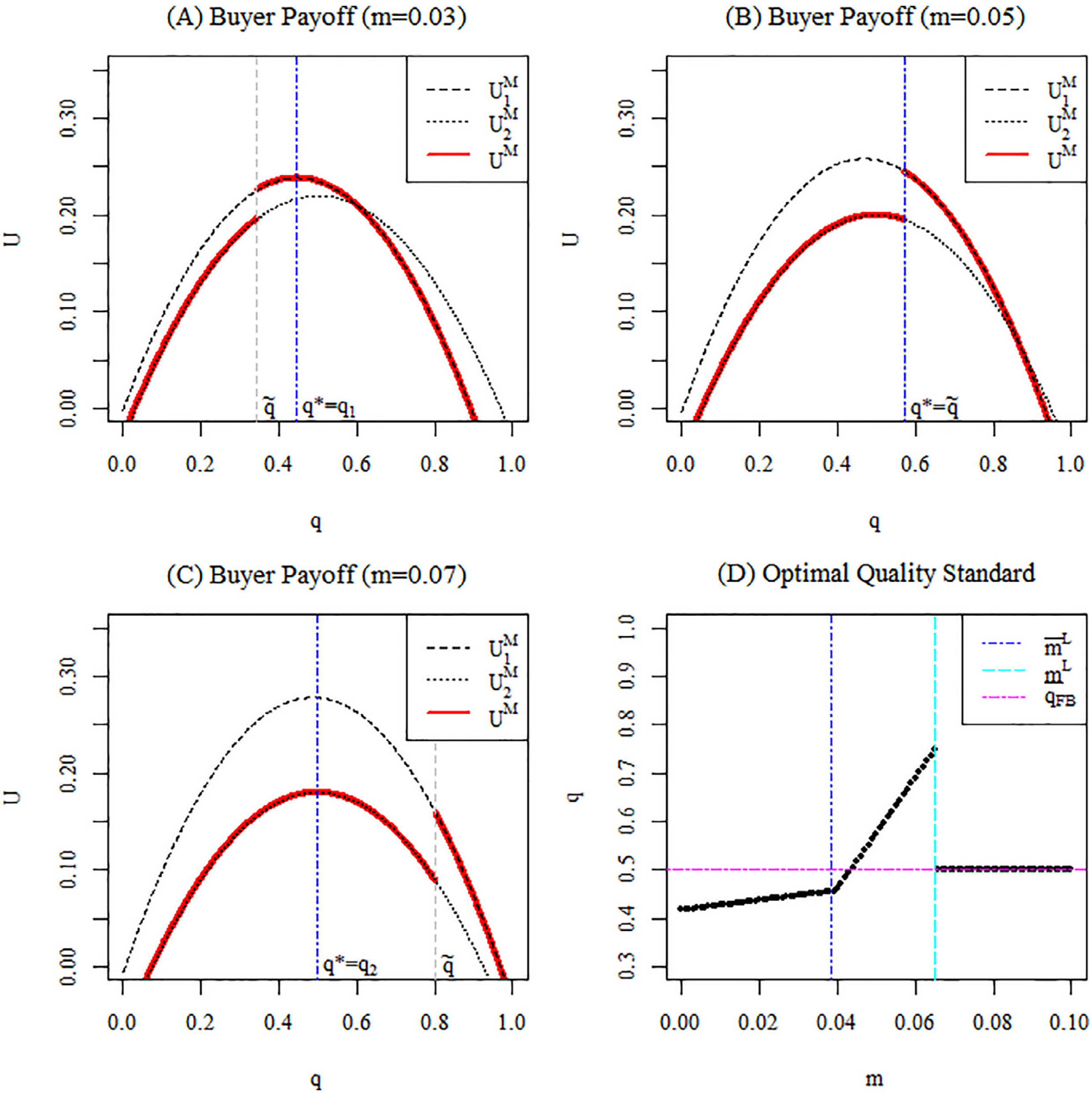

Fig. 2. Buyer payoff and optimal quality standard in M.

rent above the corruption rent, which prevents the inefficient-and-corrupt firm from winning. By selecting $\tilde{\alpha}$, the buyer pays zero rent because the two rents exactly offset one another, but it generates an upward distortion of quality. When such distortion becomes sufficiently large, it is no longer worthwhile to increase α. The buyer chooses the α_2 that allows firm 2 to win the contract rather than pay for excessively high quality.

Corollary 1. *The buyer overstates her preference for quality ($\alpha^* > 1$) for some m.*

By Eq. (3), $\tilde{\alpha}$ is determined by $m = R_T^L(\alpha)/\alpha$ and the right-hand side is increasing by Lemma 2. The threshold quality weight $\tilde{\alpha} > 1$ if $m > R_T^L(1)$. When $\alpha^* = \tilde{\alpha}$, the buyer overstates her preference for quality ($\alpha^* > 1$). Here, the condition $m > R_T^L(1)$ can be interpreted as indicating when the scope of quality manipulation is greater than the technological rent under the truthful scoring rule.

Proposition 3. *There exists $m > 0$ such that $U^L(\alpha^*) > U_{SB}^L(\alpha_{SB})$.*

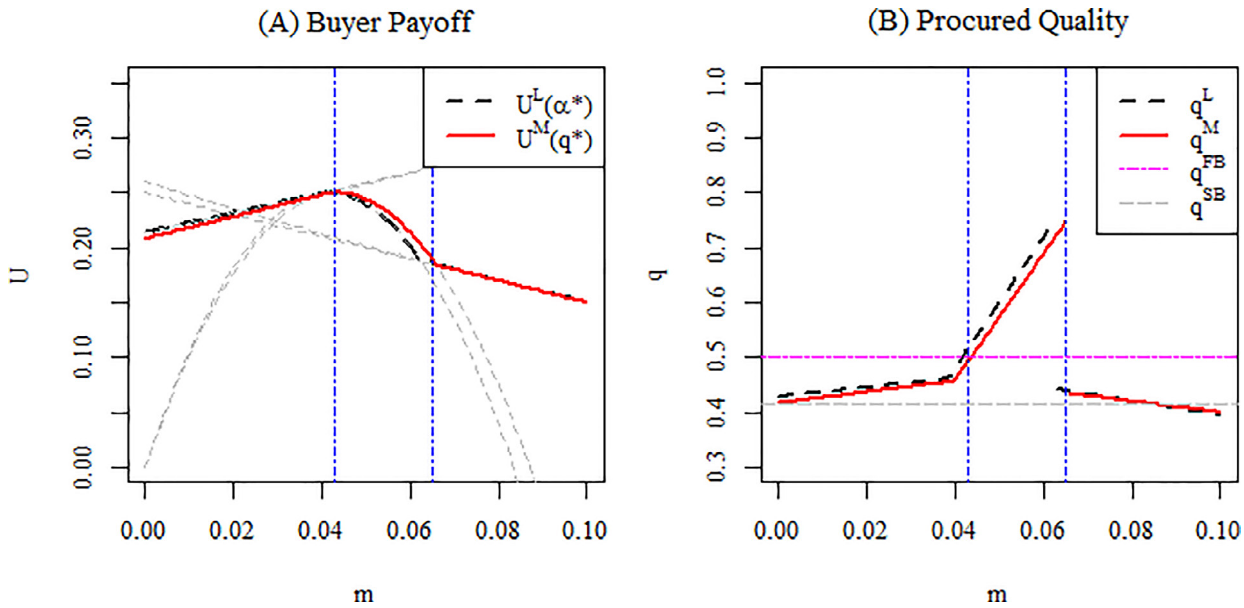

Fig. 3. Comparison of L and M. *Note*: When m lies between the two vertical lines, M performs better than L. The starting points of U^L and U^M at $m = 0$ represent the buyer's payoffs without corruption. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

This result is intuitive. $U^L_{SB}(\alpha_{SB})$ represents the buyer's payoff without corruption ($m = 0$). By (4), $U^L_1(\alpha) = q_1(\alpha) - C(q_1(\alpha), \theta_1) - R^L_T + R^L_C = U^L_{SB}(\alpha) + \alpha m$. From this expression, we see that the corruption rent erodes the technological rent. As the buyer pays less rent, her payoff increases.

2.2. Minimum-quality auction (M)

Now consider that the buyer chooses the minimum-quality auction (M) with quality standard q. When there is no quality manipulation ($m = 0$), because $C(q, \theta_1) < C(q, \theta_2)$, firm 1 wins the contract and the equilibrium prices are $p_1 = p_2 = C(q, \theta_2)$ under Bertrand-type competition. Firm 1 earns a profit $R^M_T \equiv C(q, \theta_2) - C(q, \theta_1)$, which is its technological rent under M. The buyer's payoff is $U^M_{SB}(q) = q - C(q, \theta_2)$, which has a unique maximum q_{SB}. The second-best quality q_{SB} is distorted downward compared to the first-best quality standard $q_{FB} \equiv \arg\max_q\{q - C(q, \theta_1)\}$.

When there is quality manipulation ($m > 0$), firm 2 favored by the agent can meet the minimum quality standard by delivering quality $q - m$.[20] Hence, corruption allows the inefficient-and-corrupt firm to fulfill the minimum quality at a lower cost. We define the amount of cost saving as the corruption rent, i.e., $R^M_C \equiv C(q, \theta_2) - C(q - m, \theta_2)$. When exaggerating quality, firm 2 wins the contract if $C(q - m, \theta_2) < C(q, \theta_1)$, or equivalently, $R^M_C > R^M_T$. The following lemma shows that we can find a threshold quality standard $\tilde{q}$ that determines the auction outcome.

Lemma 4. *For any $m > 0$, there exists $\tilde{q} > 0$ such that $C(q - m, \theta_2) > C(q, \theta_1)$ for $q < \tilde{q}$; $C(\tilde{q} - m, \theta_2) = C(\tilde{q}, \theta_1)$; and $C(q - m, \theta_2) > C(q, \theta_1)$ for $q > \tilde{q}$, $\tilde{q}$ increases in m.*

The buyer's payoff is a function with a discontinuity at $\tilde{q}$:

$$U^M(q) = \begin{cases} U^M_2(q) = q - m - C(q, \theta_1) & \text{for } q < \tilde{q}, \\ U^M_1(q) = q - C(q - m, \theta_2) & \text{for } q \geq \tilde{q}, \end{cases} \tag{6}$$

where the subscript indicates which firm wins the contract. The feature of the optimal quality standard is similar to the optimal quality weight.

Proposition 4. *There exist two cutoff values of the scope of quality manipulation, $\underline{m}^M$ and $\overline{m}^M$, such that*

$$q^* = \begin{cases} q_1 \equiv \arg\max_{q\in[0,\infty)} U^M_1(q) & \text{if } m < \underline{m}^M, \\ \tilde{q} & \text{if } \underline{m}^M \leq m \leq \overline{m}^M, \\ q_2 \equiv \arg\max_{q\in[0,\infty)} U^M_2(q) & \text{if } m > \overline{m}^M. \end{cases}$$

The intuition is similar to that of Proposition 2: The buyer may raise the quality standard to ensure that the efficient firm dominates the inefficient-and-corrupt firm, but doing so distorts the quality upward. When the loss from procuring

[20] To avoid negative quality, we restrict $m < q_2(\alpha)$.

exceedingly high quality outweighs the benefit from deterring corruption, the buyer allows firm 2 to win the contract. As a result, the optimal quality standard consists of three intervals, as illustrated in Fig. 2.

Corollary 2. *The buyer overstates her preference for quality ($q^* > q_{FB}$) for some m.*

When m is large enough such that $C(q_{FB}, \theta_1) < C(q_{FB} - m, \theta_2)$, then $\tilde{q} > q_{FB}$. When $q^* = \tilde{q}$ as shown in Proposition 4, the buyer requires a quality higher than the first-best level ($q^* > q_{FB}$). The condition, $C_1(q_{FB}, \theta_1) < C_2(q_{FB} - m, \theta_2)$, means that the scope of quality manipulation is sufficiently large that the efficient firm cannot win at the first-best quality.

Similar to Proposition 3, it is straightforward to show that quality manipulation can benefit the buyer because the corruption rent of the inefficient firm erodes the technological rent of the efficient firm.

Proposition 5. *There exists $m > 0$ such that $U^M(q^*) > U^M_{SB}(q_{SB})$.*

After obtaining both the optimal quality weight in L and the optimal quality standard in M, we can compare the buyer's payoffs under these two procurement mechanisms. We can show that, with quality manipulation, scoring auctions do not always dominate minimum-quality auctions.

Proposition 6. *There exists some m such that $U^M(q^*) > U^L(\alpha^*)$.*

Fig. 3 illustrates the key trade-off between corruption deterrence and quality distortion and displays the three main results. When m is small, the buyer will benefit from both rent savings and efficiency improvement because corruption deterrence requires a higher quality than the second-best level. As a result, quality becomes closer to the first-best level, and the buyer is better off than in a corruption-free environment (Propositions 3 and 5, **Result 1**).

As m becomes large, to deter corruption, the buyer must induce a higher procured quality, which eventually exceeds the first-best level (Corollaries 1 and 2, **Result 2**). Paying for such an upwardly distorted quality is costly. From Fig. 3-(B), we see that such quality distortion is less in M than in L. Because the buyer can deter corruption and procure at a lower quality level in M than in L, M outperforms L in some cases (Proposition 6, **Result 3**).

Specifically, when $m \in [\underline{m}^L, \overline{m}^L]$, by adopting L, the buyer must procure at a high quality, $q_1(\tilde{\alpha})$, from firm 1; otherwise, firm 2 can outbid firm 1 and deliver an inferior project. Under L, firm 2 has the full flexibility in selecting its price-quality combination, so it will choose $q_2(\tilde{\alpha})$ that reaches the maximum score $\bar{s}_2(\tilde{\alpha})$. However, under M, firm 2 loses flexibility in selecting quality level. When the buyer requires a quality standard $q = q_1(\tilde{\alpha})$ in M, firm 2 cannot choose $q_2(\tilde{\alpha})$ in response. This weakens firm 2, so the buyer can induce a lower quality level in M than in L, i.e., setting $q < q_1(\tilde{\alpha})$.

One caveat is that the buyer needs to know about the scope of quality manipulation (m) and cost parameters (θ_1, θ_2) to employ these results in practice. Such knowledge of the procurement environment is necessary to utilize the key trade-off and set the optimal level of the quality weight or minimum quality standard. In selecting between L and M, the buyer also needs to know whether m falls into the interval $[\underline{m}^L, \overline{m}^L]$. Therefore, the practical value of these results for procurement relies critically on the designer's knowledge and experience.

2.3. Corrupt efficient firm

We now turn to the case in which the efficient firm has a positive probability of being corrupt. The corruption relationship is formed *exogenously*.[21] Let $x \in [0, 1]$ denote the probability of the efficient firm being favored by the agent. The probability of the inefficient firm being favored is $1 - x$. The buyer knows the probability x but does not know which firm's quality is exaggerated.

When firm 1 is corrupt, it possesses both the technological rent and the corruption rent, so will win the contract for sure. In this case, the buyer is made worse off by corruption because she needs to pay both rents. If the buyer adopts L, her payoff is a function with a discontinuity at $\tilde{\alpha}$:

$$U^L(\alpha) = \begin{cases} x\left[q_1(\alpha) - C_1(\alpha) - R^L_T - R^L_C\right] + (1-x)\left[q_1(\alpha) - C_1(\alpha) - R^L_T + R^L_C\right] & \text{for } \alpha \geq \tilde{\alpha}, \\ x\left[q_1(\alpha) - C_1(\alpha) - R^L_T - R^L_C\right] + (1-x)\left[q_2(\alpha) - C_2(\alpha) + R^L_T - R^L_C\right] & \text{for } \alpha < \tilde{\alpha}. \end{cases}$$

If the buyer adopts M, her payoff is a function with a discontinuity at $\tilde{q}$:

$$U^M(q) = \begin{cases} x[q - m - C(q, \theta_2)] + (1-x)[q - C(q - m, \theta_2)] & \text{if } q \geq \tilde{q}, \\ x[q - m - C(q, \theta_2)] + (1-x)[q - m - C(q, \theta_1)] & \text{if } q < \tilde{q}. \end{cases}$$

In these two payoff functions, the terms following $(1 - x)$ are the same as (5) and (6), respectively.[22] We can show that the three main results hold as long as x is sufficiently small.

[21] In Burguet and Che (2004), this relationship is formed endogenously through a bribery competition.

[22] For both payoff functions, the terms following x are continuously decreasing in m. Hence, the optimal α and q will depend primarily on the terms following $(1 - x)$ that yield similar properties in Propositions 2 and 4, respectively.

Proposition 7. *With $x < 0.5$, under the optimal quality weight for L and the optimal quality standard for M, the procurement mechanism has the following features: (i) Corruption may benefit the buyer; (ii) the buyer may overstate her preference for quality ($\tilde{\alpha} > 1,\ \tilde{q} > q_{FB}$) ; and (iii) the buyer may obtain a higher payoff from M than L.*

Note that, for the existence of the key trade-off between corruption deterrence and quality distortion, the inefficient firm must be more likely to be favored than the efficient firm. The corruption rent erodes the technological rent only when they belong separately to the inefficient firm and the efficient firm, respectively. Otherwise, if the efficient firm is likely to possess both rents, raising the quality weight or quality standard only makes the buyer worse off.

2.4. Multiple firms

Consider the case in which there are $n > 2$ firms in the auction. Each firm's type θ is drawn independently from $F(\theta)$. We label firms by the ranking of their types, $\theta_1 < \theta_2 < \cdots < \theta_n$, where a smaller θ indicates greater efficiency. Let x_i denote the probability of firm i being favored by the agent ($\sum_{i=1}^{n} x_i = 1$).

Firm 1 is the most efficient firm. When firm 1 is favored, it will surely win and earn both the technological rent and the corruption rent. When firm $j \neq 1$ is favored, the winner depends on the relative size of the corruption rent of firm j and the technological rent of firm 1. Therefore, by treating firm j as the inefficient firm in the benchmark model, the equilibrium derived in Propositions 2 and 4 can be directly applied here. The winner will still be either the most efficient firm or the corrupt firm. Therefore, all results in Sections 2.1 and 2.2 hold in the case of multiple firms.

Note that firm 1's technological rent depends on the difference between the first-order statistic $\theta_{(1:\,n)}$ and the second-order statistic $\theta_{(2:\,n)}$, and this difference is decreasing in n. Therefore, competition reduces the efficient firm's technological rent. However, the magnitude of the corruption rent is not sensitive to the change in n.[23] Consequently, it is more difficult to deter corruption by adjusting the scoring rule (or minimum quality standard) when there are more firms. The buyer is more likely to procure from the corrupt firm and receive a project with manipulated quality. As a result, promoting competition may not reduce corruption or increase the buyer's payoff. The optimal scoring rule, auction format, and buyer's payoff will depend on the vector $\vec{x} = (x_1, x_2, \ldots, x_n)$ and the scope of corruption. In general, the effect of increasing n on the buyer's payoff is ambiguous.

There is a body of literature on the relationship between competition and corruption in procurement auctions. It is commonly believed that increasing competition is a way to reduce corruption (e.g., Ades, 1999; Coviello and Mariniello, 2014). In contrast, Celentani and Ganuza (2002) show the opposite: Competition may not reduce corruption. Li and Xu (2016) consider the case that the agent controls how many firms to invite to the auction. Depending on the specific form of bribery, corruption may or may not result in inviting fewer firms to the auction. Whether competition reduces corruption is an important policy question. In practice, there are numerous regulations intended to encourage competition, such as adopting mandatory public announcement of procurement information, requirements for a minimum number of firms, and compensation for entry. However, as shown in this paper, increasing competition reduces the efficient firm's technological rent, and thus, may make it easier for the corrupt firm to win the contract.

3. Extension and discussion

In Section 2, we assume complete information among firms on both the corruption relationship and costs. We now discuss the case of incomplete information.

3.1. Incomplete information on the corruption relationship

To analyze the case with incomplete information on the corruption relationship, we need to specify the beliefs of each firm about other firms' likelihoods of being corrupt. In some cases, the auction outcome is uncertain when the technological rent is greater than the corruption rent. The efficient firm faces two options: (i) bidding conservatively, meaning that it only outbids an honest opponent, and (ii) bidding aggressively such that it wins the contract even if the opponent is favored by the agent. The equilibrium strategy sometimes involves mixing between these two options. A detailed analysis of this case is presented below.

Consider the following modification of the two-firm model. The agent is exogenously matched to one firm and exaggerates its quality by m. The agent and the corrupt firm know that they are colluding, but the buyer and the honest firm do not. The buyer believes that firm 1 and firm 2 are favored with probability x_1 and x_2, respectively. The buyer believes that there is no corruption with probability $1 - x_1 - x_2$.[24] If firm 1 is not favored, it believes that firm 2 is favored with conditional probability $x_2/(1 - x_1)$ and that there is no corruption with probability $(1 - x_1 - x_2)/(1 - x_1) \equiv \phi$. Firm 2's beliefs can be specified in a similar way. These beliefs are important in determining the equilibrium strategy and are summarized in the following table.

[23] The corruption rent is αm under L and $C(q, \theta_j) - C(q - m, \theta_j)$ under M. Both rents do not depend on n.

[24] If $x_1 + x_2 = 0$ or $x_1 + x_2 = 1$, the model reduces to that analyzed in Section 2.

State	Buyer's belief	Firm 1's belief	Firm 2's belief
Firm 1 is favored	x_1	-	$\frac{x_1}{1-x_2}$
Firm 2 is favored	x_2	$\frac{x_2}{1-x_1}$	-
No corruption	$1-x_1-x_2$	$\frac{1-x_1-x_2}{1-x_1} \equiv \phi$	$\frac{1-x_1-x_2}{1-x_2}$

Consider that the buyer adopts a scoring auction with a linear scoring rule, $S(p,q) = \alpha q - p$. Similar to the main model, a firm's quality choice can be separated from price. Denote firm i's quality choice as $q_i = \arg\max_q \{\alpha q - C(q, \theta_i)\}$ and its equilibrium costs as $c_i = C(q_i, \theta_i)$. We use subscript $i = 1, 2$ to indicate firms and another subscript $j = 0, m$ to indicate whether firm i is honest or favored. If firm i is honest, $s_{i0} = \alpha q_i - p_{i0}$. If firm i is corrupt, $s_{im}(p_{im}) = \alpha(q_1 + m) - p_{im}$. In the equilibrium analysis, we need specify the pricing strategies of four types of firms: p_{10}, p_{1m}, p_{20}, and p_{2m}.

First, an honest firm 2 has no opportunity to win the contract and bids its cost in equilibrium, that is, $p_{20} = c_2$, under Bertrand-type competition.

Second, a corrupt firm 1 will certainly win the contract. In equilibrium, it submits a price that generates a score that matches the score of firm 2, i.e., $s_{1m} = \alpha(q_1 + m) - p_{1m} = s_{20} = \alpha q_2 - p_{20} = \alpha q_2 - c_2$. Hence, $p_{1m} = \alpha(q_1 + m) - \alpha q_2 + c_2 = c_1 + R_T^L + R_C^L$, which indicates that firm 1 earns both the technological and corruption rents as its profit.

Third, consider a corrupt firm 2. A favored firm knows its opponent is not favored. If the corruption rent dominates ($R_T^L < R_C^L$), the outcome is that firm 2 wins for certain. In this case, firm 2 will bid a price that matches firm 1's maximum score, that is, $\alpha q_2 - p_{2m} + \alpha m = \alpha q_1 - c_1$. Hence, $p_{2m} = \alpha m - \alpha q_1 + \alpha q_2 + c_1 = c_2 - R_T^L + R_C^L$. If the corruption rent is not large enough to guarantee that firm 2 will win, given firm 1's price p_{10}, a corrupt firm 2 wins the contract with a positive profit if and only if its maximum score is greater than firm 1's score, i.e., $\alpha q_2 - c_2 + \alpha m > \alpha q_1 - p_{10}$. Hence, a corrupt firm 2's best response is

$$p_{2m}(p_{10}) = \begin{cases} c_2, & \text{if } p_{10} \le c_2 + \alpha(q_1 - q_2) - \alpha m, \\ p_{10} - \epsilon, & \text{if } p_{10} > c_2 + \alpha(q_1 - q_2) - \alpha m. \end{cases} \tag{7}$$

Finally, consider an honest firm 1. There are three cases: (i) When the corruption rent dominates, firm 1 does not have a chance to win and will bid $p_{10} = c_1$ in equilibrium. (ii) When $s_{2m} > \alpha q_1 - c_1$, firm 1 cannot outbid a corrupt firm 2; it therefore selects the highest price that can outbid an honest firm 2. By matching firm 2's maximum score, $\alpha q_1 - p_{10} = \alpha q_2 - c_2$, so $p_{10}^{high} = \alpha q_1 - \alpha q_2 - c_1 + c_2 + c_1 = c_1 + R_T^L$. (iii) The case in which $s_{2m} \le \alpha q_1 - c_1$ is not trivial. From firm 1's perspective, with probability ϕ, its opponent is honest. Then firm 1 can win with certainty given its technological rent. With probability $1 - \phi$, its opponent is favored. Firm 1 can either maintain the price at p_{10}^{high}, which only wins the contract when firm 2 is honest, or outbid a corrupt firm 2 by lowering the price. Specifically, firm 1 needs to bid a price such that $\alpha q_1 - p_{10} = \alpha q_2 - p_{2m} + \alpha m$, and thus, $p_{10}^{low} = p_{2m} + \alpha(q_1 - q_2) - \alpha m$. If firm 1 sets p_{10}^{high}, it wins with probability ϕ and earns profit $\phi(p_{10}^{high} - c_1) = \phi R_T^L$. If firm 1 sets p_{10}^{low}, it wins the contract for certain and earns a profit $p_{10}^{low} - c_1$. Firm 1 will choose to bid the low price when $\phi R_T^L \le p_{10}^{low} - c_1$, that is $p_{2m} \ge R_C^L - (1 - \phi)R_T^L$. Thus, the best response of an honest firm 1 is

$$p_{10}(p_{2m}) = \begin{cases} p_{10}^{low}, & \text{if } p_{2m} \ge \alpha m - (1 - \phi)R_T^L, \\ p_{10}^{high}, & \text{if } p_{2m} < \alpha m - (1 - \phi)R_T^L. \end{cases} \tag{8}$$

The equilibrium is determined by the solution of the system of best responses (7) and (8). We can show that when the corruption rent is small ($R_T^L \le (1 - \phi)R_T^L$), an honest firm 1 can win the contract for certain. There exists a pure-strategy equilibrium in which $p_{2m} = c_2$ and $p_{10} = p_{10}^{low} = c_2 + \alpha(q_1 - q_2) - \alpha m = c_1 + R_T^L - R_C^L$. When the corruption rent is large ($R_T^L > (1 - \phi)R_T^L$), an honest firm 1 and a corrupt firm 2 cannot outbid the other for certain, and there exists no pure-strategy equilibrium. In the mixed-strategy equilibrium, p_{10} and p_{2m} follow some continuous distribution functions, $G_{10}^L(\cdot)$ and $G_{2m}^L(\cdot)$, respectively. In Burguet and Che (2004), a similar mixed-strategy equilibrium is shown when an efficient firm cannot outbid an inefficient-and-corrupt firm for certain.

In summary, the equilibrium outcome and prices are as follows.

Firm type	Outcome	Equilibrium price
Corrupt firm 1	Firm 1 wins	$p_{1m} = c_1 + R_T^L + R_C^L$
Honest firm 2	Firm 1 wins	$p_{20} = c_2$
Honest firm 1	Firm 1 wins if $R_C^L \le (1 - \phi)R_T^L$	$p_{10} = c_1 + R_T^L - R_C^L$
	Uncertain if $(1 - \phi)R_T^L < R_C^L < R_T^L$	$p_{10} \sim G_{10}^L(\cdot)$
	Firm 2 wins if $R_T^L < R_C^L$	$p_{10} = c_1$
Corrupt firm 2	Firm 1 wins if $R_C^L \le (1 - \phi)R_T^L$	$p_{2m} = c_2$
	Uncertain if $(1 - \phi)R_T^L < R_C^L < R_T^L$	$p_{2m} \sim G_{2m}^L(\cdot)$
	Firm 2 wins if $R_T^L < R_C^L$	$p_{2m} = c_2 - R_T^L + R_C^L$

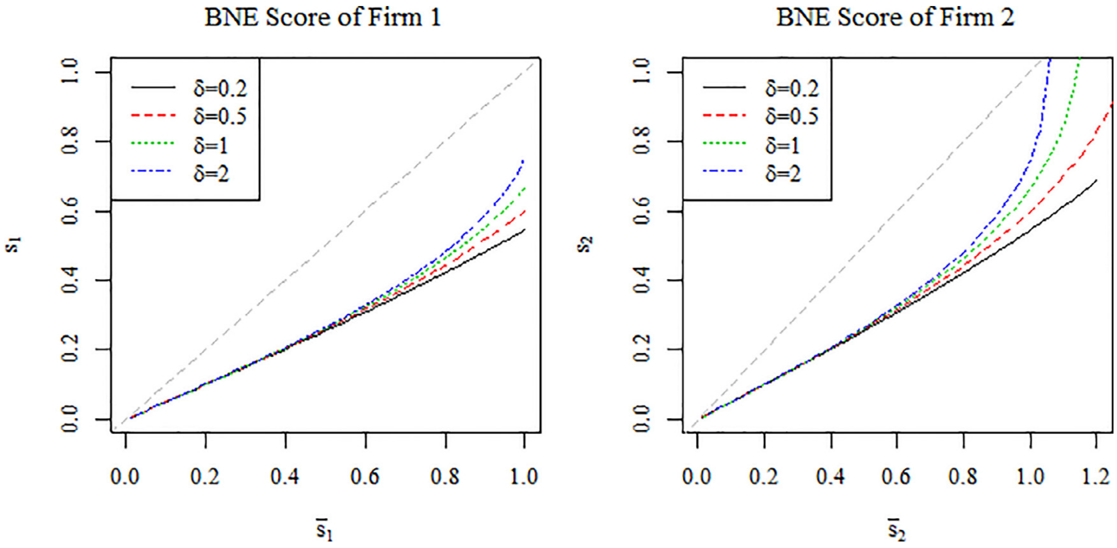

Fig. 4. Equilibrium scores in an asymmetric auction.

By a similar analysis, we can show that, in minimum-quality auctions, the outcome equilibrium prices are as follows.

Firm type	Outcome	Equilibrium price
Corrupt firm 1	Firm 1 wins	$p_{1m} = C(q, \theta_2)$
Honest firm 2	Firm 1 wins	$p_{20} = C(q, \theta_2)$
Honest firm 1	Firm 1 wins if $R_C^M \le (1-\phi)R_T^M$	$p_{10} = C(q-m, \theta_2)$
	Uncertain if $(1-\phi)R_T^M < R_C^M < R_T^M$	$p_{10} \sim G_{10}^M(\cdot)$
	Firm 2 wins if $R_T^M < R_C^M$	$p_{10} = C(q, \theta_1)$
Corrupt firm 2	Firm 1 wins if $R_C^M \le (1-\phi)R_T^M$	$p_{2m} = C(q-m, \theta_2)$
	Uncertain if $(1-\phi)R_T^M < R_C^M < R_T^M$	$p_{2m} \sim G_{2m}^M(\cdot)$
	Firm 2 wins if $R_T^M < R_C^M$	$p_{2m} = C(q, \theta_1)$

From the buyer's perspective, by adjusting the procurement mechanism, she can affect the equilibrium outcome and prices. Hence, the trade-off between corruption deterrence and quality distortion exists in the environment with incomplete information on the corruption relationship. Solving for the optimal procurement mechanism is beyond the scope of this paper. We leave it for future research.

3.2. Incomplete information on costs

Consider a modified two-firm model with incomplete information on the cost parameter θ. Two firms are *ex ante* symmetric with θ drawn from the same distribution function F. In the auction, each firm i knows its realized θ_i, the realized corruption relationship, and the distribution of the opponent's cost parameter, F. First, consider scoring auctions with the quasilinear scoring rule $S(p,q) = V(q) - p$. In this environment, Asker and Cantillon (2008) show that there is no loss of generality from assuming that each firm bids according to its pseudotype, which is the maximum score in this paper. Firm i chooses a score s_i that maximizes its expected payoff $\max_{s_i}(\bar{s}_i - s_i)\Pr(\text{win}|s_i)$.

Label the firm being favored by the agent as firm 2. The maximum scores are $\bar{s}_1 = \max_q\{V(q) - C(q,\theta_1)\}$ and $\bar{s}_2 = \max_q\{V(q+m) - C(q,\theta_2)\}$. Denote the distribution function of $\bar{s}_1$ as $H_1(s)$, which can be derived from F.[25] Denote the distribution function of $\bar{s}_2$ as $H_2(s, m)$, which depends on both F and the scope of corruption m. Quality manipulation affects the auction by shifting the distribution of firm 2's maximum score "rightward" in the sense of first-order stochastic dominance (FSD). Therefore, the auction becomes asymmetric. In general, there is no closed-form solution for asymmetric auctions, even for the case of two bidders (Krishna, 2009). We consider a special case in Kaplan and Zamir (2012) to obtain some key insights.

Suppose that the maximum score of firm 1 follows a uniform distribution, $\bar{s}_1 \sim U[0,1]$.[26] The maximum score of firm 2 follows another uniform distribution, $\bar{s}_2 \sim U[0, 1+\delta]$, where δ indicates the scope of quality manipulation. Applying the

[25] See Huang (2018) for an example.

[26] If the scoring rule is $S(p,q) = 2q - p$, $C(q,\theta) = q^2/\theta$, and $\theta \sim U[0,1]$, one can easily show that $\bar{s} = \max_q\left\{2q - q^2/\theta\right\} = \theta$. So $\bar{s} \sim U[0,1]$.

result in Kaplan and Zamir (2012), the BNE of the scoring auction is

$$\begin{cases} \mathcal{S}_1(\bar{s}_1) = \frac{(1+\delta)^2}{\delta(2+\delta)\bar{s}_1}\left[1-\sqrt{1-\frac{\delta(2+\delta)}{(1+\delta)^2}\bar{s}_1^2}\right], \\ \mathcal{S}_2(\bar{s}_2) = \frac{(1+\delta)^2}{\delta(2+\delta)\bar{s}_2}\left[\sqrt{1+\frac{\delta(2+\delta)}{(1+\delta)^2}\bar{s}_2^2}-1\right]. \end{cases}$$

There are two features of this equilibrium. First, both bidding strategies are increasing in δ. As the type distribution of firm 2 increases (in the sense of FSD), both firms bid more aggressively, as shown in Fig. 4. The buyer can reap some benefit from the intensified competition. Second, in equilibrium, firm 1 wins with probability $\Pr(1\text{ wins}|\delta) = 1/(2+2\delta)$ that decreases in δ, whereas firm 2 wins with probability $\Pr(2\text{ wins}|\delta) = (2\delta+1)/(2+2\delta)$ that increases in δ. Hence, the corrupt firm is more likely to win the contract as the scope of quality manipulation increases, and thus, the buyer is more likely to procure a project with exaggerated quality.

By adjusting the procurement mechanism, the buyer can influence the auction outcome. Setting a higher weight on quality results in a greater advantage of firm 2, which is reflected by a larger δ in this simple model. The optimal scoring rule under quality manipulation will need to strike a balance between promoting aggressive bidding and the risk of procuring an inferior project. A similar trade-off is present in selecting the quality standard in a minimum-quality auction. The nature of this trade-off is not easily captured by a tractable model due to the analytical difficulty of asymmetric auctions.

4. Conclusion

In a price-only procurement auction, the efficient firm's technological rent depends on the gap between its cost and the strongest opponent's cost. In a multi-attribute auction, the technological rent varies with the equilibrium quality choice, which in turn depends on the scoring rule. Therefore, the buyer can affect the size of the efficient firm's technological rent by adjusting the scoring rule. In this paper, we introduce quality manipulation corruption into multi-attribute auctions. A corrupt agent exaggerates a favored firm's quality and thus grants a corruption rent to this corrupt firm. An adjustment of the scoring rule now affects both the technological rent and the corruption rent. In this case, the buyer needs to adjust the procurement mechanism that to balance the trade-off between corruption deterrence and quality distortion. The optimal auction format yields three results that are distinct from those in the existing literature. First, the buyer may be better off in the presence of corruption than in a corruption-free environment. Second, the buyer may overstate her preference for quality. Third, a minimum-quality auction may perform better than the popular linear scoring auction in certain situations.

These findings offer several important policy implications. First, understanding the nature and scope of corruption is important in the procurement design problem. Lengwiler and Wolfstetter (2006) suggest reducing the weight of quality when the quality evaluation is subject to corruption. However, using a sub-optimal scoring rule not only causes direct efficiency loss but also increases the winning probability of the inefficient-and-corrupt firm. Sometimes, raising the quality weight can prevent the corrupt firm from winning and avoid the procurement of an inferior project.

Second, in most countries, public procurement is tightly regulated, and the selection of the procurement mechanism is restricted. For example, the Chinese Law of Tender requires that all high-valued government-related projects be evaluated by a three-factor weighted linear scoring rule with a capped quality weight. In this paper, we show that allowing for a large quality weight and adopting minimum-quality auctions are important tools for procurers to combat quality manipulation corruption. Regulatory policies should offer greater flexibility to buyers in procurement design (provided that the buyer is benevolent), especially in complicated environments. Using data from public procurement cases in Italy, Coviello et al. (2018) show that granting greater discretion to buyers improves procurement outcomes.

Third, in industries with a high risk of quality manipulation, eradicating all corruption would be costly and may even be impossible. In this case, allowing some less-efficient firms to be favored may make them stronger competitors and reduce the rents of large firms. However, this rent erosion can only happen when the efficient firm is honest. In reality, large and efficient firms are often those with connections and favored status. Therefore, antitrust authorities should devote more resources to monitoring large firms and harshly punish them if corruption is found. One practical method is linking the penalty to the value of contracts that the convicted firm has won in the past. Compte et al. (2005) note that preventing a rather efficient firm from bribing the agent will promote competition and benefit the buyer. Our analysis echoes this suggestion.

Finally, note that putting these policy implications into practice requires accurate knowledge of the procurement environment. To employ a desirable procurement mechanism derived in theoretical studies, buyers and regulators need to know the scope of quality manipulation and the efficiency levels of firms from both past data and experience. Therefore, recording and aggregating procurement auction data is extremely valuable for improving procurement outcomes and combating corruption.

Acknowledgments

We would like to thank Yeon-Koo Che, Fahad Khalil, Macro Haan, Jacques Lawarree, Ting Liu, Qihong Liu, Xu Tan, and Quan Wen for valuable comments and suggestions. We also thank the associate editor and two referees for their careful reading and insightful comments. The seminar participants at University of Washington, International IO Conference, Xiamen

University, Hong Kong University, Shanghai University of Finance and Economics, and Asia-Pacific IO Conference are greatly appreciated. Jijun Xia gratefully acknowledges the financial support from Shanghai Philosophy and Social Science Planning Project (#2015110470) and the SUFE Key Laboratory of Mathematical Economics.

Appendix

Proof of Lemma 1. $q_i(\alpha) = \arg\max_q \{\alpha q - C(q, \theta_i)\}$ satisfies first-order condition (FOC)

$$G \equiv \alpha - C_q(q, \theta_i) = 0. \tag{9}$$

By the implicit function theorem,

$$\frac{\partial q_i(\alpha)}{\partial \theta} = -\frac{\frac{\partial G}{\partial \theta}}{\frac{\partial G}{\partial q}} = -\frac{-C_{q\theta}}{-C_{qq}} = -\frac{C_{q\theta}}{C_{qq}} < 0.$$

Therefore, for $\theta_1 < \theta_2$, we have $q_1(\alpha) > q_2(\alpha)$.

By the envelop theorem, the maximum score is decreasing in θ,

$$\frac{d\bar{s}_i}{d\theta} = \frac{d[\alpha q - C(q, \theta)]}{d\theta} = -C_{q\theta} < 0.$$

Therefore, for $\theta_1 < \theta_2$, we have $\bar{s}_1(\alpha) > \bar{s}_2(\alpha)$. □

Proof of Lemma 2. When $m = 0$, by Lemma 1,

$$R_T^L = \alpha q_1(\alpha) - C(q_1(\alpha), \theta_1) - \alpha q_2(\alpha) + C(q_2(\alpha), \theta_2) = \bar{s}_1(\alpha) - \bar{s}_2(\alpha) > 0$$

We want to show that R_T^L is increasing and convex in α.

$$\begin{aligned}\frac{dR_T^L}{d\alpha} &= q_1(\alpha) + \alpha q_1'(\alpha) - C_q(q, \theta_1) q_1'(\alpha) - q_2(\alpha) - \alpha q_2'(\alpha) + C_q(q, \theta_2) q_2'(\alpha)\\ &= q_1(\alpha) - q_2(\alpha) + q_1'(\alpha)\underbrace{[\alpha - C_q(q, \theta_1)]}_{=0 \text{ by Lemma 1}} - q_2'(\alpha)\underbrace{[\alpha - C_q(q, \theta_2)]}_{=0 \text{ by Lemma 1}}\\ &= q_1(\alpha) - q_2(\alpha) > 0.\end{aligned} \tag{10}$$

By identity (9) and the implicit function theorem,

$$q_i'(\alpha) = -\frac{\frac{\partial G}{\partial \alpha}}{\frac{\partial G}{\partial q}} = -\frac{1}{-C_{qq}} = \frac{1}{C_{qq}} > 0.$$

Because $C_{qq\theta} > 0$, $C_{qq}(q, \theta_1) < C_{qq}(q, \theta_2)$, and thus,

$$\frac{d^2 R_T^L}{d\alpha^2} = q_1'(\alpha) - q_2'(\alpha) = \frac{1}{C_{qq}(q, \theta_1)} - \frac{1}{C_{qq}(q, \theta_2)} > 0. \tag{11}$$

□

Proof of Proposition 1. Obviously, the objective function

$$U_{SB}^L(\alpha) = q_1(\alpha) - p_1 = q_1(\alpha) - C(q_1(\alpha), \theta_1) - R_T^L$$

is continuous in α. Its first-order derivative is

$$\begin{aligned}\frac{dU_{SB}^L(\alpha)}{d\alpha} &= (1-\alpha) q_1'(\alpha) - q_1(\alpha) + \alpha q_2'(\alpha) + q_2(\alpha) - C_q(q, \theta_2) q_2'(\alpha)\\ &= (1-\alpha) q_1'(\alpha) - q_1(\alpha) + q_2(\alpha) + q_2'(\alpha)\underbrace{[\alpha - C_q(q, \theta_2)]}_{=0 \text{ by Lemma 1}}\\ &= (1-\alpha) q_1'(\alpha) - q_1(\alpha) + q_2(\alpha).\end{aligned}$$

$$q_i''(\alpha) = \frac{d}{d\alpha}\left(\frac{1}{C_{qq}(q_i(\alpha), \theta)}\right) = -\frac{C_{qqq}}{C_{qq}^2} q_i'(\alpha) = -\frac{C_{qqq}}{C_{qq}^3}.$$

Its second-order derivative is

$$\begin{aligned}\frac{d^2 U_{SB}^L(\alpha)}{d\alpha^2} &= (1-\alpha) q_1''(\alpha) - q_1'(\alpha) - q_1'(\alpha) + q_2'(\alpha)\\ &< (1-\alpha) q_1''(\alpha) - q_1'(\alpha)\end{aligned}$$

$$= -(1-\alpha)\frac{C_{qqq}}{C_{qq}^3} - \frac{1}{C_{qq}} < 0$$

$$\Leftrightarrow C_{qqq} > -\frac{C_{qq}^2}{1-\alpha}, \quad \text{for all } \alpha \in [0,1] \tag{12}$$

Note that,

$$U_{SB}^{L'}(0) = \frac{1}{C_{qq}(q,\theta_1)} - \underbrace{q_1(0)}_{=0} + \underbrace{q_2(0)}_{=0} > 0,$$

$$U_{SB}^{L'}(1) = 0 - q_1(1) + q_2(1) < 0.$$

Because $U_{SB}^{L'}(\alpha)$ is decreasing in [0,1], there exists a unique maximum $\alpha_{SB} \in (0,\,1)$ that satisfies $U_{SB}^{L'}(\alpha) = 0$. □

Proof of Lemma 3. Define $\xi(\alpha) = \bar{s}_1(\alpha) - \bar{s}_2(\alpha)$. It has first-order derivative

$$\xi'(\alpha) = q_1(\alpha) - q_2(\alpha) - m.$$

Because $q_1(0) = q_2(0) = 0$, when $m > 0$, $\xi'(0) < 0$. The second-order derivative of $\xi(\alpha)$ is

$$\xi''(\alpha) = q_1'(\alpha) - q_2'(\alpha),$$

which is positive for all α by (11).

Combining the condition that $\xi(0) = 0$, $\xi'(0) < 0$, and $\xi''(\alpha) > 0$, we conclude that there exists a unique $\tilde{\alpha} > 0$ such that $\xi(\alpha) < 0$, for $\alpha < \tilde{\alpha}$; $\xi(\tilde{\alpha}) = 0$; and $\xi(\alpha) > 0$, for $\alpha > \tilde{\alpha}$.

$\xi'(\alpha)$ starts at a negative value at $\alpha = 0$ and increases in α. At the intersection $\xi(\tilde{\alpha}) = 0$, $\xi(\alpha)$ must have a positive slope. Therefore,

$$\xi'(\tilde{\alpha}) = q_1(\tilde{\alpha}) - q_2(\tilde{\alpha}) - m > 0. \tag{13}$$

By the implicit function theorem,

$$\frac{d\tilde{\alpha}}{dm} = -\frac{-\tilde{\alpha}}{q_1(\tilde{\alpha}) - q_2(\tilde{\alpha}) - m} = \frac{\tilde{\alpha}}{q_1(\tilde{\alpha}) - q_2(\tilde{\alpha}) - m} > 0. \tag{14}$$

□

Proof of Proposition 2. (i) Property of U_1^L

By Proposition 1, $U_{SB}^L(\alpha)$ is strictly concave

$$U_1^L(\alpha) = q_1(\alpha) - C(q_1(\alpha),\theta_1) - R_T^L + R_C^L = U_{SB}^L(\alpha) + \alpha m.$$

$$\frac{d^2U_1^L(\alpha)}{d\alpha^2} = \frac{d^2U_{SB}^L(\alpha)}{d\alpha^2} < 0.$$

Therefore, $U_1^L(\alpha)$ is strictly concave and has a unique maximum $\alpha_1 \equiv \arg\max_{\alpha\in[0,\infty)} U_1^L(\alpha)$. $U_1^L(\cdot)$ is increasing on $[0,\,\alpha_1]$ and is decreasing on $[\alpha_1,\,\infty)$.

(ii) Property of U_2^L

$U_2^L(\cdot)$ has a bounded domain, $U_2^L(0) = 0$, and $U_2^L(\tilde{\alpha}) = q_2(\tilde{\alpha}) - C(q_2(\tilde{\alpha}),\theta_2) < \infty$, so there exists $\alpha_2 \equiv \arg\max_{\alpha\in[0,\tilde{\alpha}]} U_2^L(\alpha)$. For $\alpha \in [0,\tilde{\alpha}]$, $R_T^L \le R_C^L$, we have

$$\begin{aligned} U_1^L(\alpha) - U_2^L(\alpha) &= q_1(\alpha) - C(q_1(\alpha),\theta_1) - R_T^L + R_C^L - \left[q_2(\alpha) - C(q_2(\alpha),\theta_2) + R_T^L - R_C^L\right] \\ &= \underbrace{q_1(\alpha) - C(q_1(\alpha),\theta_1) - [q_2(\alpha) - C(q_2(\alpha),\theta_2)]}_{>0} + 2\underbrace{(R_C^L - R_T^L)}_{\ge 0} > 0. \end{aligned}$$

This implies that for all $\alpha \in [0,\tilde{\alpha}]$, which is the domain of $U_2^L(\cdot)$, $U_2^L(\alpha) < U_1^L(\alpha)$.

(iii) Compare α_1 and $\tilde{\alpha}$

Because $U_1^L(\alpha) = U_{SB}^L(\alpha) + \alpha m$, we can show that

$$\frac{dU_1^L(\alpha)}{d\alpha} = \frac{dU_{SB}^L(\alpha)}{d\alpha} + m = 0, \text{ and } \frac{d\alpha_1}{dm} = -\frac{1}{\frac{d^2U_{SB}^L(\alpha)}{d\alpha^2}} > 0,$$

so α_1 increases linearly in m. By Eq. (14), we can obtain that

$$\frac{d^2\tilde{\alpha}}{dm^2} = \frac{\alpha}{[q_1(\alpha) - q_2(\alpha) - m]^2} > 0,$$

so $\tilde{\alpha}$ is increasing and convex in m. Hence, there exists a unique $\underline{m}^L$ such that for $m < \underline{m}^L$, $\tilde{\alpha} < \alpha_1$; for $m > \underline{m}^L$, $\tilde{\alpha} > \alpha_1$; and $\tilde{\alpha} = \alpha_1$ when $m = \underline{m}^L$.

(iv) When $m < \tilde{m}^L$, $\alpha^* = \alpha_1$.

In this case, U_1^L can reach its maximum α_1 and $U_1^L(\tilde{\alpha}) < U_1^L(\alpha_1)$. By (ii), for all $\alpha \in [0, \tilde{\alpha}]$, which is the domain of $U_2^L, U_2^L(\alpha) < U_1^L(\alpha) < U_1^L(\tilde{\alpha}) < U_1^L(\alpha_1)$. Hence, the maximum is at α_1.

(v) When $m > \underline{m}^L$, $\tilde{\alpha} > \alpha_1$, α^* is either at $\tilde{\alpha}$ or α_2.

By (ii), for $\alpha_2 \in [0, \tilde{\alpha}]$, $U_2^L(\alpha_2) < U_1^L(\alpha_2) \le U_1^L(\alpha_1)$. This is because U_1^L is decreasing on $[\alpha_1, \infty)$ and $\lim_{\alpha\to\infty} U_1^L(\alpha) = -\infty$. There exists a unique $\check{\alpha} \in [\alpha_1, \infty)$ such that $U_2^L(\alpha_2) = U_1^L(\check{\alpha})$. Moreover, if $\tilde{\alpha} < \check{\alpha}, U_2^L(\alpha_2) < U_1^L(\tilde{\alpha})$, $\alpha^* = \tilde{\alpha}$; and if $\tilde{\alpha} > \check{\alpha}$, $U_2^L(\alpha_2) > U_1^L(\tilde{\alpha})$, $\alpha^* = \alpha_2$.

Because $\tilde{\alpha}$ increases in m, we can find $\overline{m}^L$ that equates $\tilde{\alpha}$ and $\check{\alpha}$. For $m < \overline{m}^L$, $\tilde{\alpha} < \check{\alpha}$, $\alpha^* = \tilde{\alpha}$; for $m > \overline{m}^L$, $\tilde{\alpha} > \check{\alpha}$, $\alpha^* = \alpha_2$. □

Proof of Proposition 3. When the buyer chooses the quality weight optimally, U^L is continuous on $m \in [0, \underline{m}^L)$ and $\alpha^* = \alpha_1$.

$U_1^L(\alpha) = U_{SB}^L(\alpha) + \alpha m$. As $m \to 0$, $U_1^L(\alpha) \to U_{SB}^L(\alpha)$ and $\alpha_1 \to \alpha_{SB}$.

$$\lim_{m\to 0} \frac{\partial U_{SB}^L(\alpha)}{\partial \alpha}\bigg|_{\alpha=\alpha_1} = \lim_{m\to 0} \frac{\partial U_{SB}^L(\alpha)}{\partial \alpha}\bigg|_{\alpha=\alpha_{SB}} = 0,$$

$$\begin{aligned}\frac{\partial U_1^L(\alpha_1)}{\partial m} &= \frac{\partial}{\partial m}\left\{U_{SB}^L(\alpha_1) + \alpha_1 m\right\} \\ &= \frac{\partial U_{SB}^L(\alpha)}{\partial \alpha}\bigg|_{\alpha=\alpha_1} \frac{d\alpha_1}{dm} + m\frac{d\alpha_1}{dm} + \alpha_1.\end{aligned}$$

Therefore, we have

$$\lim_{m\to 0} \frac{\partial U^L}{\partial m} = \alpha_{SB} > 0.$$

The continuous function U^L has a strictly positive slope at $m = 0$. This is sufficient to guarantee that the proposition holds. □

Proof of Lemma 4. By the single-crossing property $C_{q\theta} > 0$ and $C(0, \theta) = 0$, $C(q, \theta_1)$ and $C(q, \theta_2)$ only cross once at $q = 0$. $C(q - m, \theta_2)$ is a rightward parallel shift of $C(q, \theta_2)$, and thus, $C(q - m, \theta_2)$ and $C(q, \theta_1)$ will also only cross once. Denote this positive crossing point as $\tilde{q}$, which makes $C(\tilde{q}, \theta_1) = C(\tilde{q} - m, \theta_2)$.

Because $C_{q\theta} > 0$, for $\theta_1 < \theta_2$, $C_q(q, \theta_1) < C_q(q, \theta_2)$. By the property of parallel shifting, $C_q(q, \theta_2) = C_q(q - m, \theta_2)$, therefore, $C_q(q, \theta_1) < C_q(q - m, \theta_2)$. Because $C(q, \theta_1)$ is flatter than $C(q - m, \theta_2)$ and they cross at $\tilde{q}$, obviously, for $q > \tilde{q}$, $C(q, \theta_1) < C(q - m, \theta_2)$; for $q < \tilde{q}$, $C(q, \theta_1) > C(q - m, \theta_2)$.

The threshold $\tilde{q}$ is determined by $C(\tilde{q}, \theta_1) - C(\tilde{q} - m, \theta_2) = 0$. By the implicit function theorem,

$$\frac{d\tilde{q}}{dm} = \frac{C_q(\tilde{q} - m, \theta_2)}{C_q(\tilde{q} - m, \theta_2) - C_q(\tilde{q}, \theta_1)} > 0. \tag{15}$$

□

Proof of Proposition 4.

(i) U_1^M and U_2^M are both strictly concave.

$$\frac{dU_1^M(q)}{dq} = 1 - C_q(q - m, \theta_2), \quad \frac{dU_1^M(q)}{dq} = -C_{qq}(q - m, \theta_2) < 0.$$

$$\frac{dU_2^M(q)}{dq} = 1 - C_q(q, \theta_1), \quad \frac{dU_2^M(q)}{dq} = -C_{qq}(q, \theta_1) < 0.$$

Therefore, there is a unique maximum for U_1^M and U_2^M, denoted as $q_1 \equiv \arg\max_{q\in[0,\infty)} U_1^M(q)$and $q_2 \equiv \arg\max_{q\in[0,\infty)} U_2^M(q)$, respectively.

(ii) For any $q \in [0, \tilde{q}]$, $U_1^M(q) > U_2^M(q)$, because

$$U_1^M(q) - U_2^M(q) = \underbrace{C(q, \theta_1) - C(q - m, \theta_2)}_{\ge 0 \text{ for } q\le\tilde{q}} + m > 0.$$

(iii) Compare q_1 and $\tilde{q}$.

Because q_1 is determined by $1 - C_q(q - m, \theta_2)$,

$$\frac{dq_1}{dm} = -\frac{-C_{qq}(q - m, \theta_2)(-1)}{-C_{qq}(q - m, \theta_2)} = 1. \tag{16}$$

Hence, q_1 increases linearly in m. By (15), $\tilde{q}$ is increasing and convex in m. Therefore, there exists a unique $\underline{m}^M$ such that for $m < \underline{m}^M$, $\tilde{q} < q_1$; for $m > \underline{m}^M$, $\tilde{q} > q_1$; and $\tilde{q} = q_1$ when $m = \underline{m}^M$.

(iv) When $m < \underline{m}^M$, $\tilde{q} < q_1$, U_1^M can reach its maximum q_1 and $U_1^L(\tilde{q}) < U_1^L(q_1)$. By (ii), for all $q \in [0, \tilde{q}]$, which is the domain of U_2^M, $U_2^M(q) < U_1^M(q) < U_1^M(\tilde{q}) < U_1^M(q_1)$. Hence, the maximum is at q_1.

(v) When $m > \underline{m}^M$, $\tilde{q} > q_1$, q^* is either at $\tilde{q}$ or q_2.

By (ii), $U_2^M(\tilde{q}) < U_1^M(\tilde{q})$, so U^M jumps upward at the discontinuity point. This is because U_1^M is decreasing on $[q_1, \infty)$ and $\lim_{q\to\infty} U_1^L(q) = -\infty$.

Hence, there exists a unique $\check{q} \in [q_1, \infty)$ such that $U_2^M(q_2) = U_1^M(\check{q})$. Because q_2 does not depend on m and $\tilde{q}$ increases with m, there exists a unique $\overline{m}^M$ that equates $\tilde{q}$ and $\check{q}$. For $m < \overline{m}^M$, $\tilde{q} < \check{q}$, $U_2^M(q_2) < U_1^M(\tilde{q})$, so $q^* = \tilde{q}$; for $m > \overline{m}^M$, $\tilde{q} > \check{q}$, $U_2^M(q_2) > U_1^M(\tilde{q})$, so $q^* = q_2$. □

Proof of Proposition 5. When the buyer chooses the quality standard optimally, U^M is continuous on $m \in [0, \underline{m}^M)$ and $q^* = q_1$. Differentiate $U_1^M(q) = q_1 - C(q_1 - m, \theta_2)$ with respect to m:

$$\frac{\partial U_1^M(q_1)}{\partial m} = \frac{dq_1}{dm} - C_q\left(\frac{dq_1}{dm} - 1\right) = C_q > 0.$$

By (16), $\frac{dq_1}{dm} = 1$, so

$$\frac{\partial U_1^M(q_1)}{\partial m} = 1 - C_q(1 - -1) = 1 > 0$$

The continuous function U^L has a strictly positive slope at $m = 0$. This is sufficient to guarantee that the proposition holds. □

Proof of Proposition 6. Consider the level of m such that $q^* = \tilde{q} = q_{FB}$ and $\alpha^* = \tilde{\alpha}$. This implies that $C(q_{FB}, \theta_1) = C(q_{FB} - m, \theta_2)$. In this case, the buyer's payoff from adopting M is

$$U^M(q^*) = U^M(\tilde{q}) = q_{FB} - C(q_{FB} - m, \theta_2) = q_{FB} - C(q_{FB}, \theta_1).$$

Because $R_T^L = R_C^L = \tilde{\alpha} m$, the buyer's payoff from adopting L is

$$U^L(\alpha^*) = U^L(\tilde{\alpha}) = q_1(\tilde{\alpha}) - C_1(\tilde{\alpha}).$$

The payoff difference is

$$U^M(q^*) - U^L(\alpha^*) = q_{FB} - C(q_{FB}, \theta_1) - q_1(\tilde{\alpha}) + C_1(\tilde{\alpha}) \geq 0.$$

The difference is positive because $q_{FB} - C(q_{FB}, \theta_1) \geq q - C(q, \theta_1)$ by the definition of q_{FB}. As long as $q_1(\tilde{\alpha}) \neq q_{FB}$, the inequality is strict, that is, $U^M(q^*) > U^L(\alpha^*)$. □

Proof of Proposition 7. Example 1 is sufficient to guarantee that the three main results hold in the case of $x > 0$. □

Example 1. Consider the cost function $C(q, \theta_i) = \theta_i q^2$. Set firm 1's efficiency parameter as $\theta_1 = 1$ and firm 2's efficiency parameter as $\theta_2 = \theta > 1$, i.e., $C(q, \theta_1) = q^2$, $C(q, \theta_2) = \theta q^2$. (In generating Figs. 1–3, we set $\theta = 1.2$ and $x = 0$.)

Under L, by Lemma 3, $q_1(\alpha) = \frac{\alpha}{2}$, $q_2(\alpha) = \frac{\alpha}{2\theta}$. Equilibrium costs are $C_1(\alpha) = \frac{\alpha^2}{4}$ and $C_2(\alpha) = \frac{\alpha^2}{4\theta}$. Firm 1's technological rent is $R_T^L \equiv \alpha q_1(\alpha) - C_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) = \frac{\alpha^2}{4}\left(\frac{\theta-1}{\theta}\right)$. When $m = 0$, the buyer's payoff is $U_{SB}^L(\alpha) = q_1(\alpha) - C_1(\alpha) - R_T^L = \frac{\alpha}{2} - \frac{\alpha^2}{4}\left(\frac{2\theta-1}{\theta}\right)$. The optimal quality weight is $\alpha_{SB} = \frac{\theta}{2\theta-1} < 1$. The maximized payoff is $U_{SB}^L(\alpha_{SB}) = \frac{1}{4}\left(\frac{\theta}{2\theta-1}\right)$.

When $m > 0$, the corruption rent is $R_C^L = \alpha m$. By equating the two rents, we obtain the threshold quality weight $\tilde{\alpha}(m) = \frac{4\theta m}{\theta-1}$. The buyer's payoff function is

$$U^L(\alpha) = \begin{cases} \frac{\alpha}{2} - \frac{\alpha^2}{4}\left(\frac{2\theta-1}{\theta}\right) + (1-2x)\alpha m & \text{if } \alpha \geq \frac{4\theta m}{\theta-1}, \\ x\left[\frac{\alpha}{2} - \frac{\alpha^2}{4}\left(\frac{2\theta-1}{\theta}\right)\right] + (1-x)\left[\frac{\alpha}{2\theta} - \frac{\alpha^2}{4}\left(\frac{2-\theta}{\theta}\right)\right] - \alpha m & \text{if } \alpha < \frac{4\theta m}{\theta-1}. \end{cases}$$

The peaks of U_1^L and U_2^L are reached at $\alpha_1 = \frac{\theta(1+2m-4xm)}{2\theta-1}$ and $\alpha_2 = \frac{x\theta-x-2\theta m+1}{3x\theta-3x-\theta+2}$, respectively.

Using M, when $m = 0$, the buyer's payoff is $U_{SB}^M(q) = q - C(q, \theta_2) = q - \theta q^2$. The optimal quality standard is $q_{SB} = \frac{1}{2\theta}$, and the maximum payoff is $U_{SB}^M(q_{SB}) = \frac{1}{4\theta}$. Compared to L, one can easily show that $U_{SB}^M(q_{SB}) < U_{SB}^L(\alpha_{SB})$ for $\theta > 1$.

When $m > 0$, firm 1's technological rent is $R_T^M = C(q, \theta_2) - C(q, \theta_1) = \theta q^2 - q^2 = (\theta - 1)q^2$ and firm 2's corruption rent (provided that it is corrupt) is $R_C^M = \theta q^2 - \theta(q - m)^2 = \theta(2qm - m^2)$. By equating the two rents, the threshold quality $\tilde{q} = \frac{m(\theta+\sqrt{\theta})}{\theta-1}$. The buyer's payoff function is

$$U^M(q) = \begin{cases} q - m - \theta q^2 + 2(1-x)\theta m q - (1-x)\theta m^2 & \text{if } q \geq \frac{m(\theta+\sqrt{\theta})}{\theta-1}, \\ q - m - x\theta q - (1-x)q^2 & \text{if } q < \frac{m(\theta+\sqrt{\theta})}{\theta-1}. \end{cases}$$

The peaks of U_1^M and U_2^M are reached at $q_1 = \frac{1+2(1-x)\theta m}{2\theta}$ and $q_2 = \frac{(1-x\theta)}{2(1-x)}$, respectively. The procurement quality in L is $q_1(\tilde{\alpha}) = \frac{2\theta m}{\theta-1} > q_{FB}$ ($\tilde{\alpha} > \alpha_{FB}$), when $m > \frac{\theta-1}{4\theta}$. In M, $\tilde{q} = \frac{m(\theta+\sqrt{\theta})}{\theta-1} > q_{FB}$ when $m > \frac{\theta-1}{2(\theta+\sqrt{\theta})} > \frac{\theta-1}{4\theta}$. Therefore, for large m, procurement quality is distorted less in M than in L.

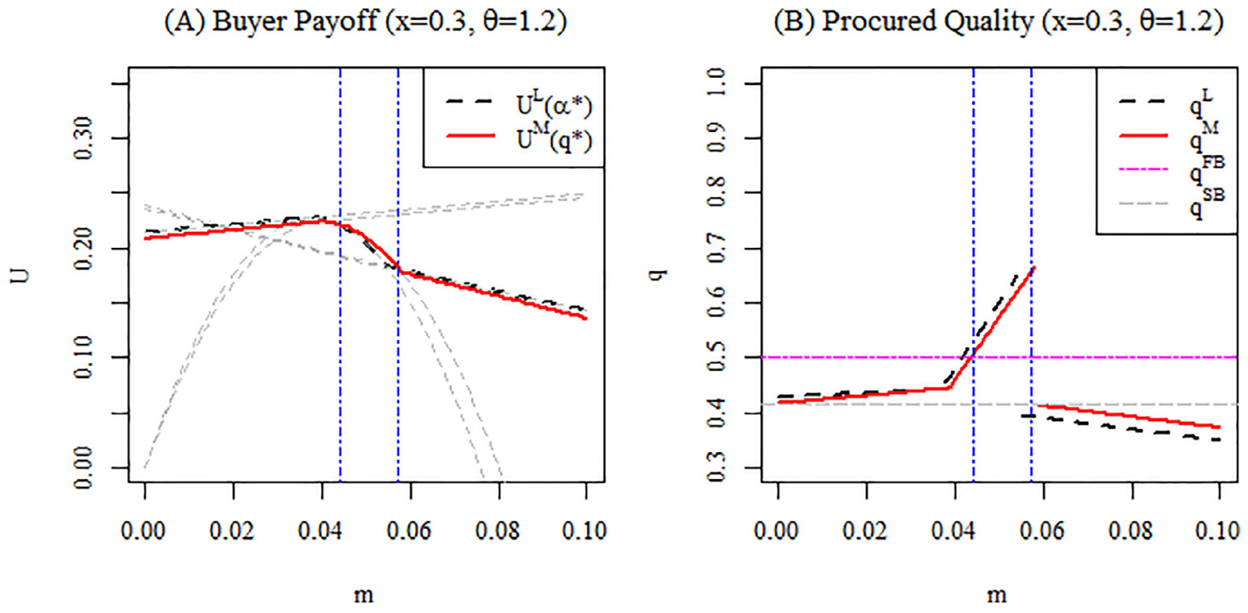

Fig. 5. Comparison of L and M.

We can easily verify the three main results using this example.

(1) $m^{L*} = \arg\max_{m\geq 0} U^L(\alpha^*(m,x)) > 0$ and $m^{M*} = \arg\max_{m\geq 0} U^M(q^*(m,x)) > 0$; (2) the buyer may overstate her preference for quality, causing the induced procurement quality to be higher than the first-best level; and (3) for $x < 0.5$, there exists an interval $(\underline{m}, \overline{m})$ such that for $m \in (\underline{m}, \overline{m})$, $U^L(\alpha^*(m, x)) < U^M(q^*(m, x))$.

Restricted by the length of the paper, we show the analytical solution when $x = 0$. The case with $x > 0$ is illustrated in Fig. 5.

When $x = 0$, the optimal quality weight is

$$\alpha^* = \begin{cases} \alpha_1 = \frac{\theta(1+2m)}{2\theta-1}, & \text{for } 0 \leq m \leq \frac{\theta-1}{6\theta-2}, \\ \tilde{\alpha} = \frac{4\theta m}{\theta-1}, & \text{for } \frac{\theta-1}{6\theta-2} < m \leq \bar{m}^L, \\ \alpha_2 = \frac{1-2\theta m}{2-\theta}, & \text{for } \bar{m}^L < m \leq \frac{1}{2\theta}. \end{cases}$$

The buyer's payoff under L is

$$U^L(\alpha^*(m,0)) = \begin{cases} U_1^L(\alpha_1) = \frac{\theta(1+2m)^2}{4(2\theta-1)}, & \text{for } 0 \leq m \leq \frac{\theta-1}{2\theta(\sqrt{\theta}+1)}, \\ U_1^L(\tilde{\alpha}) = \frac{2\theta(\theta-1)m-4\theta^2m^2}{(\theta-1)^2}, & \text{for } \frac{\theta-1}{6\theta-2} < m \leq \bar{m}^L, \\ U_2^L(\alpha_2) = \frac{(1-2\theta m)^2}{4\theta(2-\theta)}, & \text{for } \bar{m}^L < m \leq \frac{1}{2\theta}, \\ 0 & \text{for } m > \frac{1}{2\theta}. \end{cases}$$

The cutoff value $\bar{m}^L$ can be found by solving $U_2^L(\alpha_2) = U_1^L(\tilde{\alpha})$. Here, $\bar{m}^L = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where $a = 12\theta^4 - 24\theta^3 - 4\theta^2$, $b = -8\theta^4 + 28\theta^3 - 24\theta^2 + 4\theta$, $c = -(\theta-1)^2$.

The optimal quality standard is

$$q^* = \begin{cases} q_1 = \frac{1+2\theta m}{2\theta} & \text{for } 0 \leq m \leq \frac{\theta-1}{2(\theta+\sqrt{\theta})}, \\ \tilde{q} = \frac{m(\theta+\sqrt{\theta})}{\theta-1}, & \text{for } \frac{\theta-1}{2(\theta+\sqrt{\theta})} < m \leq \bar{m}^M, \\ q_2 = \frac{1}{2}, & \text{for } \bar{m}^M < m \leq \frac{1}{4}. \end{cases}$$

$$U^M(q^*(m,0)) = \begin{cases} U_1^M(q_1) = \frac{1}{4\theta} + m, & \text{for } 0 \leq m \leq \frac{\theta-1}{2(\theta+\sqrt{\theta})}, \\ U_1^M(\tilde{q}) = \frac{(\theta+\sqrt{\theta})m}{\theta-1} - \frac{\theta(\sqrt{\theta}+1)^2m^2}{(\theta-1)^2}, & \text{for } \frac{\theta-1}{2(\theta+\sqrt{\theta})} < m \leq \bar{m}^M, \\ U_2^M(q_2) = \frac{1}{4} - m, & \text{for } \bar{m}^M < m \leq \frac{1}{4}, \\ 0 & \text{for } m > \frac{1}{4}. \end{cases}$$

The cutoff value $\bar{m}^M$ can be found by solving $U_2^M(q_2) = U_1^M(\tilde{q})$. Here, $\bar{m}^M = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where $a = -\frac{\theta(\sqrt{\theta}+1)^2}{(\theta-1)^2}$, $b = \frac{2\theta+\sqrt{\theta}-1}{\theta-1}$, and $c = -\frac{1}{4}$.

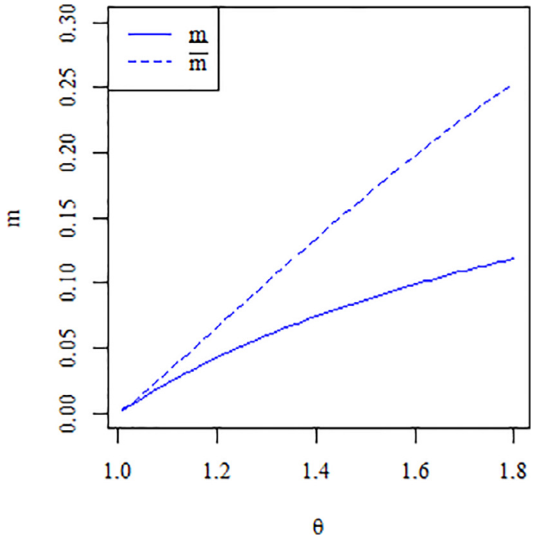

Fig. 6. The interval that M outperforms L.

Fig. 5-(A) illustrates these two payoff functions when $x = 0.3$ and $\theta = 1.2$. We see that when $m \in (\underline{m}, \overline{m}), U^L(\alpha^*) < U^M(q^*)$. Here, $\underline{m} = \frac{(\theta-1)(\theta-\sqrt{\theta})}{3\theta^2-2\theta\sqrt{\theta}-\theta}$, and $\overline{m} = \frac{-b+\sqrt{b^2-4ac}}{2a}$, where $a = 4\theta^2\left[(\theta-1)^2 + (\sqrt{\theta}+1)^2(2-\theta)\right]$, $b = -4\theta(\theta+\sqrt{\theta})(\theta-1)(2-\theta) - 4\theta(\theta-1)^2$, and $c = (\theta-1)^2$. Fig. 6 shows that when the efficiency difference (measured by θ) increases, the interval $(\underline{m}, \overline{m})$ expands.

References

Ades, A., 1999. Rents, competition, and corruption. Am. Econ. Rev. 89 (4), 982–993.
Albano, G.L., Cesi, B., Iozzi, A., 2017. Public procurement with unverifiable quality: the case for discriminatory competitive procedures. J. Public Econ. 145, 14–26.
Aryal, G., Gabrielli, M.F., 2013. Testing for collusion in asymmetric first-price auctions. Int. J. Ind. Org. 31 (1), 26–35.
Asker, J., 2010. A study of the internal organization of a bidding cartel. Am. Econ. Rev. 100 (3), 724–762.
Asker, J., Cantillon, E., 2008. Properties of scoring auctions. RAND J. Econ. 39 (1), 69–85.
Asker, J., Cantillon, E., 2010. Procurement when price and quality matter. RAND J. Econ. 41 (1), 1–34.
Athey, S., Levin, J., Seira, E., 2011. Comparing open and sealed bid auctions: evidence from timber auctions. Q. J. Econ. 126 (1).
Bajari, P., Ye, L., 2003. Deciding between competition and collusion. Rev. Econ. Stat. 85 (4), 971–989.
Branco, F., 1997. The design of multidimensional auctions. RAND J. Econ. 28 (1), 63–81.
Burguet, R., 2017. Procurement design with corruption. Am. Econ. J. Microecon. 9 (2), 315–341.
Burguet, R., Che, Y.-K., 2004. Competitive procurement with corruption. RAND J. Econ. 50–68.
Burguet, R., Perry, M.K., 2009. Preferred suppliers in auction markets. RAND J. Econ. 40 (2), 283–295.
Burguet, R., Perry, M.K., 2014. Preferred Suppliers in Asymmetric Auction Markets. Working Paper Series. Barcelona GSE.
Celentani, M., Ganuza, J.-J., 2002. Corruption and competition in procurement. Eur. Econ. Rev. 46 (7), 1273–1303.
Che, Y.-K., 1993. Design competition through multidimensional auctions. RAND J. Econ. 24 (4), 668–680.
Che, Y.-K., 2006. Procurement. In: The New Palgrave Dictionary of Economics.
Chen-Ritzo, C.-H., Harrison, T.P., Kwasnica, A.M., Thomas, D.J., 2005. Better, faster, cheaper: an experimental analysis of a multiattribute reverse auction mechanism with restricted information feedback. Manag. Sci. 51 (12), 1753–1762.
Compte, O., Lambert-Mogiliansky, A., Verdier, T., 2005. Corruption and competition in procurement auctions. RAND J. Econ. 36 (1), 1–15.
Coviello, D., Guglielmo, A., Spagnolo, G., 2018. The effect of discretion on procurement performance. Manag. Sci 64 (2) Pages v-x, 495-981.
Coviello, D., Mariniello, M., 2014. Publicity requirements in public procurement: evidence from a regression discontinuity design. J. Public Econ. 109, 76–100.
David, E., Azoulay-Schwartz, R., Kraus, S., 2006. Bidding in sealed-bid and english multi-attribute auctions. Decis. Support Syst. 42 (2), 527–556.
Dini, F., Pacini, R., Valletti, T., 2006. Scoring rules. In: Handbook of Procurement, pp. 293–321.
Engelbrecht-Wiggans, R., Haruvy, E., Katok, E., 2007. A comparison of buyer-determined and price-based multiattribute mechanism. Market. Sci. 629–641.
Fugger, N., Katok, E., Wambach, A., 2015. Collusion in dynamic buyer-determined reverse auctions. Manag. Sci. 62 (2), 518–533.
Graham, D.A., Marshall, R.C., Richard, J.-F., 1990. Differential payments within a bidder coalition and the Shapley value. Am. Econ. Rev. 80 (3), 493–510.
Gretschko, V., Wambach, A., 2016. Procurement under public scrutiny: auctions versus negotiations. RAND J. Econ. 47 (4), 914–934.
Hanazono, M., Nakabayashi, J., Tsuruoka, M., 2015. Procurement Auctions with General Price-quality Evaluation. Working Paper.
Hendricks, K., Porter, R., Tan, G., 2008. Bidding rings and the winner's curse. RAND J. Econ. 39 (4), 1018–1041.
Huang, Y., 2018. Detecting Quality Manipulation Corruption in Scoring Auction. Working Paper.
Kaplan, T.R., Zamir, S., 2012. Asymmetric first-price auctions with uniform distributions: analytic solutions to the general case. Econ. Theory 50 (2), 269–302.
Krishna, V., 2009. Auction Theory. Academic Press.
Kuhn, S., Sherman, L.B., 2014. Curbing Corruption in Public Procurement: A Practical Guide. Transparency International.
Laffont, J.-J., Tirole, J., 1991. The politics of government decision-making: a theory of regulatory capture. Q. J. Econ. 106 (4), 1089–1127.
Lengwiler, Y., Wolfstetter, E., 2006. Corruption in procurement auctions. In: Handbook of Procurement, pp. 412–451.
Li, D. Z., Xu, M., 2016. Competition in Procurement Auctions with Corruption. Working Paper.
McAfee, R., McMillan, J., 1992. Bidding rings. Am. Econ. Rev. 82 (3), 579–599.
Nishimura, T., 2015. Optimal design of scoring auctions with multidimensional quality. Rev. Econ. Des. 19 (2), 117–143.

Pesendorfer, M., 2000. A study of collusion in first-price auctions. Rev. Econ. Stud. 67 (3), 381–411.
Porter, R.H., Zona, J.D., 1993. Detection of bid rigging in procurement auctions. J. Polit. Econ. 101 (3), 518–538.
Porter, R.H., Zona, J.D., 1999. Ohio school milk markets: an analysis of bidding. RAND J. Econ. 30 (2), 263–288.
Takahashi, H., 2018. Strategic design under uncertain evaluations: structural analysis of design-build auctions. RAND J. Econ. 49 (3), 594–618.
Tian, G., Liu, C., 2008. Limited corruption in sealed-bid auctions. Econ. Res. J. 3, 130–141.
Tran, A., 2011. Which Regulations Reduce Corruption? Evidence from the Internal Records of a Bribe-paying Firm. Working Paper.
Yoganarasimhan, H., 2015. Estimation of beauty contest auctions. Market. Sci. 35 (1), 27–54.