Financial Fraud and Investor Awareness

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Keywords: financial fraud, investor naivety, unawareness, shrouding

JEL Classification: D14, D83, G11

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1 Introduction

Financial fraud refers to firms taking deceptive actions to exploit investors, such as Ponzi schemes and running away with the money. The existence of a large number of financially "illiterate" investors (Lusardi and Mitchell, 2014) opens the door for financial fraud because these investors are likely to be attracted by products that offer too-good-to-be-true returns.[1] Misleading product descriptions may induce naive investors to underestimate default risk and purchase products that are not consistent with their risk attitudes. The spread of financial fraud suggests that many naive investors may be unaware of the possibility of such fraud. To prevent firms from exploiting these naive investors, policymakers may employ regulatory policies in financial markets such as interest rate ceilings,[2] restrictions on product design,[3] and minimum legislative standards for firms.[4] However, excessive regulations may limit the product choices of investors and possibly reduce welfare. Therefore, the level of sophistication possessed by general investors is an important factor in determining whether certain regulations are necessary. After the 2008 financial crisis, the question of how to strike a balance between protecting investors and respecting investors' own decisions has received considerable attention in policy discussions (Campbell et al., 2011; Campbell, 2016).

Given this background, we build a model with firm(s) strategically choosing whether to exploit naive investors by offering financial products with too-high-to-be-true returns. There is a fraction of naive investors who are unaware that a firm can commit financial fraud. Specifically, they do not know that the firm can seize the return on their investment, and thus underestimate the true risk of a fraudulent financial product. Therefore, naive investors' investment decisions are inconsistent with their risk attitudes. Their behaviors, in turn, create an incentive for the firm to commit financial fraud.

If policymakers can reduce the proportion of naive investors through an education program, they can compel the firm to behave honestly. Moreover, if this proportion drops below a certain threshold, the firm will not offer fraudulent products even if doing so is costless because of the fear of losing the majority of sophisticated investors. Note that, after becoming aware of possible financial fraud, risk-averse investors will completely avoid products with

[1] A few papers have attempted to theoretically study the link between financial literacy and investment decisions, as well as the (mis-)selling behaviors of financial professionals in the market (e.g., Lusardi et al., 2017).

[2] Many countries have imposed interest rate ceilings (`https://en.wikipedia.org/wiki/Interest_rate_ceiling`). See Modigliani and Sutch (1966) for a discussion.

[3] For example, the Investment Company Act of 1940 strictly regulates the structure of mutual funds, imposing severe restrictions on liabilities and complex capital structures. See Campbell et al. (2010) for an extensive discussion.

[4] One example of such policy is the SAFE Mortgage Licensing Act passed in 2008. See `https://mortgage.nationwidelicensingsystem.org/SAFE/Pages/default.aspx`

too-high-to-be-true returns, but some highly risk-seeking investors may still take a gamble even though the net expected return is negative.

After studying the monopoly case, we introduce an entrant firm to the market and consider a leader-follower model. With competition, three types of equilibrium may arise depending on the costs of committing financial fraud. When the costs are high, both firms offer normal products, but their profits will be driven down to zero due to price (rate of return) competition. When the costs are low, both firms offer fraudulent products with returns at the interest rate ceiling, and they share the market for naive investors. Between these cases, the market may fall into a separating equilibrium: One firm offers a high-return fraudulent product that attracts all naive investors, while the other firm sells a normal product to all sophisticated investors. In this case, both firms earn positive profits. As a result, if the market leader offers a normal product in a monopoly, investors' welfare may be harmed after the follower's entry, because naive investors will be exploited by the fraudulent product in the separating equilibrium. Interestingly, we show that the leader has a stronger incentive to commit financial fraud than in the monopoly case, because a competing entrant firm makes it less attractive to offer a normal product.

Next, we study firms' private incentive to disclose or unshroud information as well as its policy implications. Suppose that each firm can costlessly disclose the information about the possibility of financial fraud, which reduces the proportion of naive investors. We find a trade-off in the case of separating equilibrium: The honest firm has the incentive to increase the proportion of sophisticated investors to obtain a larger market share. However, it does NOT want to increase this proportion too much because if exploiting naive investors becomes unprofitable, the other firm will deviate from offering a fraudulent product and start competing for sophisticated investors. Under this trade-off, while lowering the interest rate ceiling makes the fraudulent product less attractive, it may not be welfare-improving because the honest firm may decide to conceal information, which prevents the dishonest firm from competing in the market for normal products. Similarly, increasing legal punishment and implementing a public education program also discourage the honest firm from disclosing information.

Our paper contributes to the growing literature on bounded rationality and its applications to contracting problems and public policy.[5] When making decisions, investors may be unaware of important factors such as add-on pricing (Kosfeld and Schüwer, 2017), hidden contract terms (Agarwal et al., 2017), certain options (Auster and Pavoni, 2018), economic

[5]Relatedly, in the moral hazard literature, researchers have also studied the cases in which the agent is unaware of some of his own actions (von Thadden and Zhao, 2012) and some possible states (Auster, 2013). When contracts are incomplete, small shocks can cause significant asset-price volatility, which may lead to severe financial fraud. For an intensive literature review, see Allen and Gale (2005).

states (Lei and Zhao, 2020), and the financial product issuer's informational advantage (Kondor and Kőszegi, 2017). In many markets, firms exploit naive consumers through hidden add-on prices or surcharges (e.g., Gabaix and Laibson, 2006), complex pricing schemes (e.g., Carlin, 2009), steep marginal charges (e.g., Grubb, 2009), deceptive products (e.g., Heidhues et al., 2017), or automatic renewal (e.g., Murooka and Schwarz, 2018).[6] A similar mechanism of competition exacerbating dishonest behaviors may also exist in these markets because competition can make the market for sophisticated consumers less attractive. In addition, honest firms may also exhibit limited incentive of unshrouding if a separating equilibrium emerges. Our paper differs from the existing literature by considering a sequential-move model in which the follower can observe the leader's action and hence will have strategic responses. We also assume that consumers may have heterogeneous risk preferences, which is in contrast to most of the existing research. Consequently, we find that having a follower in the market will increase the leader's incentive to commit financial fraud, and limit his willingness to disclose information even if doing so is costless.

The paper proceeds as follows. In Section 2 we introduce a stylized model that captures the interaction between a firm and a representative investor who is possibly unaware that the firm may commit financial fraud. Section 3 extends our model to a leader-follower environment and studies each firm's incentive to disclose information and implications of policy instruments. Section 4 concludes. Some proofs are relegated to Appendix C.

2 Baseline Model

A firm needs outside finance $I > 0$ to fund a risky project that may succeed with probability p and fail with probability $1 - p$. If the project succeeds, it generates a positive cash return $\omega = R$ for the firm. If it fails, the return is normalized to $\omega = 0$. The outcome of the project ω is publicly observable. A representative investor (he) has Bernoulli utility $u = m^\alpha$, where m is the net payoff in a contingency. The risk preference parameter α is distributed over $(0, +\infty)$ with a commonly known c.d.f. $F(\alpha)$ and a strictly positive p.d.f. $f(\alpha)$.[7] We assume that the distribution of α satisfies the standard monotone likelihood ratio property (MLRP), namely, $f(\alpha)/[1 - F(\alpha)]$ is nondecreasing in α.

The firm offers a financial product as a contract that specifies a monetary return $r \leq \bar{R}$ repaid to the investor when $\omega = R$ and zero otherwise. The upper bound $\bar{R}$ represents the

[6] In parallel to the behavioral economics literature differentiating naive and sophisticated consumers, an alternative modeling approach assumes that consumers differ in their costs of acquiring information such as search costs (e.g., Armstrong et al., 2009).

[7] Our utility function modifies the isoelastic utility function, and $\alpha > 1$ represents risk-seeking preferences.

interest rate ceiling permitted by law, which is assumed to be sufficiently high.[8] In addition to r, the firm also chooses whether to behave honestly ($x = 0$) or commit financial fraud ($x = 1$). If the firm chooses $x = 0$, it repays r to the investor as stated in the contract when $\omega = R$. We call the contract with $x = 0$ a normal product. If the firm chooses $x = 1$, it refuses to repay r when the project succeeds ($\omega = R$) but instead repays $\underline{r} \in [0, R]$. We call the contract with $x = 1$ a fraudulent product. Providing a fraudulent product incurs a cost $c > 0$ to the firm, which can be interpreted as the expected reputation loss or legal punishment. $\underline{r}$ is exogenously given, which consists of the fixed assets or intellectual property created by the project that the firm cannot reap from the fraud.[9] Therefore, the firm's private benefit from committing financial fraud is $R - \underline{r}$ when the project succeeds. Suppose that $p\underline{r} < I < p(R-\underline{r})$. The first inequality means that the project is not profitable if the firm defaults, and the second inequality induces the risk-neutral firm to operate the project before taking away the money.

The timing of the game is depicted in Figure 1 and described as follows. The firm first chooses x and offers a contract with repayment r to the investor. If the investor rejects the offer, both parties receive zero payoffs and the game ends. If the investor accepts the offer, ω is realized and observed. Given that $\omega = R$, the firm repays r if $x = 0$ and repays $\underline{r}$ if $x = 1$. Note that the firm decides whether to commit financial fraud before the realization of the project outcome, because it involves certain preparations before offering the fraudulent product to the investor.[10]

Figure 1: Timeline in the Baseline Model

Our model differs from the existing literature in several aspects. First, in the literature of shrouded attributes and hidden add-ons (Gabaix and Laibson, 2006; Heidhues et al., 2017), the product and the add-on cannot be treated as two separate products. For example, a

[8]Specifically, we need the interest rate ceiling to be non-binding for the firm offering a normal product.

[9]An alternative explanation is that $\underline{r}$ may represent the fraudulent product's future discounted value in a dynamic setting. Risk-seeking investors may want to gamble for this return or resale this product to others even when the product is fraudulent; this is likely to be observed in pyramid or Ponzi schemes.

[10]For example, the firm has to prepare exaggerated marketing materials to attract potential investors and/or provide a forged auditor's report to deceive the regulators. Detailed descriptions of these preparations can be found in many court cases. See, e.g., United States of America vs. Robert A. Stanford (`im.ft-static.com/content/images/b6eabf92-b631-11e1-8ad0-00144feabdc0.pdf`).

printer cannot be used without an ink cartridge. The key problem in the add-on pricing literature is how to price a single product. However, in the present paper, the normal product and the fraudulent product are two distinct financial assets. Because we assume that investors are heterogeneous in their risk preferences, the fraudulent product may be preferred by risk-loving ones even if they are rational. Second, in the literature of inferior-quality products, the quality differences are exogenous, and low-quality products are by assumption not preferable (Armstrong and Chen, 2009). In our model, the normal product and the fraudulent product differ only in their corresponding firms' willingness to make promised repayments. The return of the normal product is endogenously determined by firms, so it is not necessarily that the normal product strictly dominates the fraudulent one.

When the investor observes that the firm is behaving honestly ($x = 0$), he accepts the firm's offer if $pr^\alpha \geq I^\alpha$. Hence, when $r > I$, the product is purchased by an investor of type α if

$$\alpha \geq \log_{\frac{r}{I}} \frac{1}{p} \equiv \underline{\alpha}(r).$$

The lower threshold $\underline{\alpha}(r)$ is decreasing in p and r and increasing in I, which implies that an investor is more likely to invest in a project with lower default risk, a higher rate of return, and less initial investment.

An honest firm chooses r to maximize its expected profit, i.e., $p(R-r)[1-F(\underline{\alpha}(r))]$. The first-order condition is

$$-p[1-F(\underline{\alpha}(r))] + \frac{p(R-r)\log_{\frac{r}{I}}\frac{1}{p}}{r\ln\frac{r}{I}} f(\underline{\alpha}(r)) = 0. \tag{1}$$

When $r \to I$, the left-hand side of (1) goes to $+\infty$; when $r = R$, the left-hand side of (1) becomes negative. Hence, under the MLRP, there is an interior solution $r^* \in (I, R)$ that yields a profit $p(R-r^*)[1-F(\underline{\alpha}(r^*))]$ to the firm. We assume $\underline{r}$ is sufficiently small and $\bar{R}$ is sufficiently large so that $\underline{r} < r^* < \bar{R}$. If $\bar{R} \leq r^*$, the firm will simply choose $\bar{R}$ since its profit function is nondecreasing when $r \in (0, r^*)$. Note that setting $\bar{R} < r^*$ hurts investors' welfare as the binding interest rate ceiling prevents the firm from offering its optimal product and distorts the market.

2.1 Investor awareness

Suppose that there are two types of investors in the population. A fraction $\lambda \in (0,1)$ of investors are *naive*: They are unaware that the firm has the option $x = 1$. In other words, they mistakenly believe that the firm can only choose $x = 0$ and do not have financial fraud in mind.[11] The remaining fraction $1-\lambda$ of investors are *sophisticated*, as they are fully aware of and can observe the firm's action x.[12]

Our assumption of investor awareness is motivated by findings in Gui et al. (2020). In this paper, they experimentally measure investors' risk preferences and present them with a hypothetical investment decision question. They find that a large fraction of investors make investment decisions that are inconsistent with their risk preferences, implying that they may be unaware of the high risk associated with high-return products.

For expositional clarity, let

$$\pi^0 = p(R-r^*)[1-F(\underline{\alpha}(r^*))],\ \pi_s^1 = p(R-\underline{r})[1-F(\underline{\alpha}(\underline{r}))],\ \pi_n^1 = p(R-\underline{r})[1-F(\underline{\alpha}(\bar{R}))].$$

Here, π^0 is the firm's revenue when it provides a normal product ($x=0$); π_s^1 is the firm's revenue when it provides a fraudulent product ($x=1$) to sophisticated investors; and π_n^1 is the firm's revenue when it provides a fraudulent product ($r=\bar{R}, x=1$) to exploit naive investors.

Because $\underline{\alpha}(r)$ is decreasing in r and $\underline{r} < r^* < \bar{R}$, when $\underline{r}$ is sufficiently small and $\bar{R}$ is sufficiently large, we have $\pi_s^1 < \pi^0 < \pi_n^1$. From the firm's perspective, selling a fraudulent product to sophisticated investors is less profitable than selling a normal product, but the latter is less profitable than selling a fraudulent product to exploit naive investors. Note that, while $p\underline{r} < I$, some risk-seeking and sophisticated investors will purchase the product as long as $\underline{r} > I$. As $\underline{r} \to I$, $\underline{\alpha}(\underline{r}) \to +\infty$.

Let (r, x) denote the firm's pure strategy. From our previous analysis, when the firm chooses $x=0$, it will offer $r=r^*$ to maximize its expected profit. When the firm chooses $x=1$, it will propose $r=\bar{R}$ to attract as many naive investors as possible. Therefore, the firm plays $(r^*, 0)$ if

$$\pi^0 \geq \lambda\pi_n^1 + (1-\lambda)\pi_s^1 - c \ \Leftrightarrow\ c \geq \lambda\pi_n^1 + (1-\lambda)\pi_s^1 - \pi^0 \equiv c^*.$$

The results above are summarized in Proposition 1.

Proposition 1. *There exists c^* such that:*

[11] We assume that naive investors are unaware that they may be unaware. That is, when they see the firm offering an unreasonably high rate of return, such as $r=\bar{R}$, they will simply take it as a mistake without further reasoning. Some other works, e.g., Chung and Fortnow (2016); Tirole (2009, 2016); Zhao (2015), assume that agents recognize that their cognitive ability is limited and that their understanding of the game could be incorrect. In other words, agents are aware of their unawareness.

[12] This assumption resembles that in the add-on pricing literature. In Gabaix and Laibson (2006), naive consumers are unaware of part of the price that they need to pay when making the purchasing decision, but sophisticated consumers can anticipate the add-on price. On the empirical side, Brown et al. (2010) find that, for online auctions, there are indeed naive consumers underestimating the hidden shipping charges.

a). When $c \geq c^*$, there exists an equilibrium in which the firm plays $(r^*, 0)$, i.e., it offers a normal product with a rate of return r^*; both types of investors with $\alpha \geq \underline{\alpha}(r^*)$ accept the contract.

b). When $c \leq c^*$, there exists an equilibrium in which the firm plays $(\bar{R}, 1)$, i.e., it offers a fraudulent product with a rate of return $\bar{R}$; sophisticated investors with $\alpha \geq \underline{\alpha}(\underline{r})$ accept the contract, and naive investors with $\alpha \geq \underline{\alpha}(\bar{R})$ accept the contract.

Note that, if the firm proposes $r = r^*$, the default risk is $1-p$; if the firm proposes $r = \bar{R}$, the default risk is 1. Therefore, a high-return financial product is associated with a high risk of default as a result of financial fraud.

It is also instructive to compute investors' welfare W when the firm offers different products. When a normal product is provided, the representative investor's welfare is

$$W_N^m = \int_{\underline{\alpha}(r^*)}^{+\infty} [p(r^*)^\alpha - I^\alpha] dF(\alpha), \tag{2}$$

where the superscript m represents "monopoly". When a fraudulent product is provided, investors' welfare is

$$W_F^m = \lambda \int_{\underline{\alpha}(\bar{R})}^{+\infty} [p\underline{r}^\alpha - I^\alpha] dF(\alpha) + (1-\lambda) \int_{\underline{\alpha}(\underline{r})}^{+\infty} [p\underline{r}^\alpha - I^\alpha] dF(\alpha). \tag{3}$$

Since when $\alpha < \underline{\alpha}(\underline{r})$, $p\underline{r}^\alpha - I^\alpha < 0$, we have $W_F^m < W_N^m$.

2.2 Education program

Based on Proposition 1, we can study the effect of a financial education program that reduces the fraction of naive investors. Suppose that the equilibrium is characterized by (b) of Proposition 1. Our first observation is that, when $p\bar{R} \geq I$, risk-averse investors are more sensitive to the education program. Before the education program, naive investors with $\alpha \geq \underline{\alpha}(\bar{R})$ purchase the fraudulent product, which consists of all the risk-seeking investors and some of the risk-averse investors since $p\bar{R} > I$, $\underline{\alpha}(\bar{R}) < 1$. However, after the education program, only those with $\alpha \geq \underline{\alpha}(\underline{r})$ will still invest in the fraudulent product. Since $p\underline{r} < I$, $\underline{\alpha}(\underline{r}) > 1$, risk-averse investors never buy the fraudulent product after education, while some risk-seeking investors still purchase it. We state this result in Proposition 2.

Proposition 2. *Suppose that the equilibrium is characterized by (b) of Proposition 1, and $p\bar{R} \geq I$. The education program prevents risk-averse investors from investing in the fraudulent product with probability* 1 *and prevents risk-seeking investors from investing in the* *fraudulent product with probability* $\frac{1-F(\alpha(r))}{1-F(1)}$*. Therefore, risk-averse investors are more sensitive to the education program.*

Proposition 2 is consistent with the experimental and survey results in Gui et al. (2020), in which they elicit the driving force for investors to purchase fraudulent financial products. They show that an eye-opening education program significantly reduces investors' tendency to invest in risky products, especially among those who are risk-averse.

Our second observation is that the education program not only directly prevents some of the naive investors from being mis-sold, but also indirectly reduces or even eliminates financial fraud. Note that a firm commits financial fraud if the cost c is below the cutoff c^*, and this cutoff is increasing in λ. Therefore, having more naive investors triggers the firm to offer a fraudulent financial product. In contrast, if the education program reduces λ, thereby lowering the cutoff cost c^*, then the firm will more often find it less worthwhile to commit financial fraud. Moreover, when λ is sufficiently low, i.e.,

$$\lambda \leq \frac{\pi^0 - \pi_s^1}{\pi_n^1 - \pi_s^1} \equiv \lambda^*,$$

c^* becomes nonpositive, and thus the firm has no incentive to commit financial fraud at all. This result is summarized in Proposition 3, with graphical illustration in Figure 2.

Proposition 3. *c^* increases with λ, and there exists λ^* such that when when $\lambda \leq \lambda^*$, $c^* \leq 0$. That is, a decrease in λ implies a decrease in the firm's incentive to commit financial fraud. When λ is sufficiently small, the firm has no incentive to commit financial fraud irrespective of c.*

Following Propositions 1 and 2, the comparative statics result in Proposition 3 leads to a common result in the behavioral contracting literature: Education not only prevents some naive investors from being exploited but also reduces the firm's incentive to offer a fraudulent product.[13] The firm is more likely to behave honestly in a market with a larger proportion of sophisticated investors.[14]

[13]This self-reinforcing pattern is similar to the results in von Thadden and Zhao (2012) and Li et al. (2016), who find that a monopolistic firm is more likely to reveal the adverse effects of its product in a market with fewer naive consumers. Relatedly, Zhou (2008) suggests that naive consumers tend to overestimate the importance of certain attributes of a product after firms' advertising and finds that increasing the proportion of naive consumers will also reduce the surplus of sophisticated consumers. Schumacher and Thysen (2017) provide a synthetic model with this self-reinforcing pattern in a more general setting.

[14]Our result in Proposition 3 is in line with Jin and Leslie (2003) offering empirical evidence that mandatory information disclosure increases restaurants' hygiene scores by making consumers aware of the health risks. That is, information disclosure may have both direct effects (i.e., it increases consumers' awareness of health risks) and indirect effects (i.e., as more consumers become aware of health risks, restaurants have higher incentive to improve their hygiene scores).

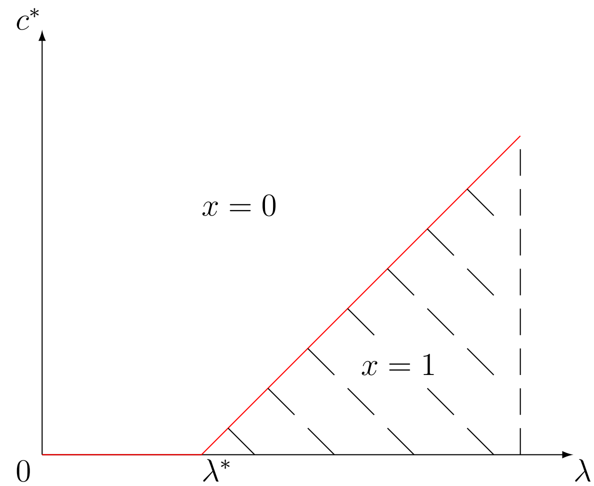

Figure 2: Graphical illustration of Proposition 3

3 Leader-follower Model

There is mounting evidence that the reputation or credibility of financial intermediaries depends on their historical performance, so well-established firms may suffer huge reputation loss after their financial misconduct gets detected (Chemmanur and Fulghieri, 1994; Gopalan et al., 2011; Griffin et al., 2014). Besides, CEOs and board members get financial penalties or reputation losses after financial fraud, and these penalties are related to firm-specific factors such as firm size and corporate governance (Agrawal et al., 1999; Fich and Shivdasani, 2007). Therefore, well-established firms may have a larger cost of committing financial fraud, because they either have good performance records in the past, or attract substantially more attention from regulatory authorities, while newcomers are likely to care less about their reputation. In this section, we model the incumbent firm as the market leader with a (possibly) higher cost of committing financial fraud, and introduce an entrant acting as the follower that can observe the leader's action and respond strategically.[15] We are also interested in their incentive to disclose information about the possibility of financial fraud. The analysis provides several insights into regulation and policies.

Consider the same environment as in the baseline model except that now there are two firms in the market: a leader and a follower. The two firms may differ in their costs of committing financial fraud but are identical in all other respects. In particular, we denote

[15] The leader-follower setting echos the observation that the financial sector is usually less competitive, and there are several large firms or banks acting as market leaders. For example, Nathan and Neave (1989) find no decreasing trend in asset concentrations in Canada's financial system. Bikker and Haaf (2002) show that the banking industry is highly concentrated in countries like Denmark, Greece, Netherlands, and Switzerland.

the leader as firm H, and the follower as firm L. Let c_j be firm j's cost of committing financial fraud, and assume that $0 < c_L \leq c_H$.

In this section, we take firm L's entry as given and assume that entry is costless. In Appendix A we study a variant of our model where there is a positive entry cost, and firm L makes its entry decision endogenously after observing firm H's move. Interestingly, we find that there is no NE when the entry cost is positive, and firm H's incentive to offer a fraudulent product is also affected by the entry decision.

Investors are wealth-constrained, so each investor can purchase only one product. This assumption implies that, *ceteris paribus*, investors will purchase whichever product offers a higher rate of return. If an investor is indifferent between two products, he purchases each product with equal probability.

The timing of the game is now depicted in Figure 3 and described as follows. First, firm H chooses whether to commit financial fraud (x_H) and the rate of return of its product (r_H). Observing firm H's decision, firm L chooses whether to commit financial fraud (x_L) and the rate of return of its product (r_L). Then, investors decide whether and which product to purchase. Finally, the outcomes of both projects are realized and payments are made according to contracts.

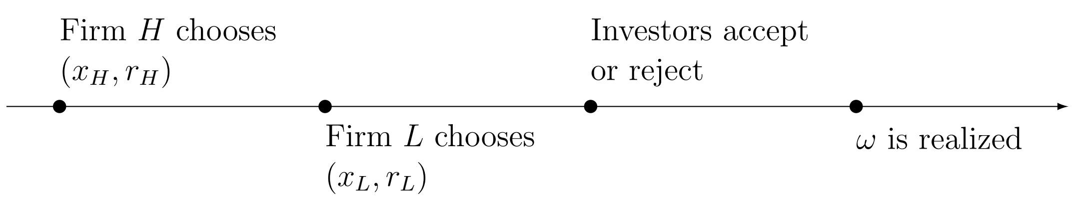

Figure 3: Timeline in the Leader-follower Model

Depending on c_H and c_L, there are several types of equilibrium based on x_H and x_L. In a *normal-product equilibrium* (NE), both firms offer normal products ($x_H = x_L = 0$). In a *fraudulent-product equilibrium* (FE), both firms commit financial fraud ($x_H = x_L = 1$). Finally, in a *separating equilibrium* (SE), one firm provides a normal product, while the other firm commits financial fraud.[16] For simplicity, we focus on the case with a nonbinding interest rate ceiling, i.e., $\bar{R} > R$.

Proposition 4 summarizes our results in the leader-follower environment.

Proposition 4. *a). When c_L is large, there exists an NE in which both firms offer a normal product with $r = R$.*

[16] Relatedly, Armstrong et al. (2009) studies prominent firms in search markets. In this paper, if consumers differ in their search costs, a prominent firm will serve the entire market of consumers with infinite cost of searching, while other firms will share the market of consumers with zero search cost.

b). When c_H is small, there exists an FE in which both firms offer a fraudulent product with $r = \bar{R}$.

c). Otherwise, there exists an SE in which one of the firms offers a normal product, while the other firm offers a fraudulent product with $r = \bar{R}$.

Figures 4 and 5 illustrate two cases for the results of Proposition 4 depending on a cutoff

$$\bar{\lambda} = \frac{\pi_s^1}{\pi_n^1 + \pi_s^1 - 2\pi^0}.$$

Suppose $\lambda \geq \bar{\lambda}$. Then there exist c_H^* and c_L^* such that: (i) If $c_L \geq c_L^*$, there exists an NE; (ii) If $c_H \leq c_H^*$, there exists an FE; (iii) If $c_H \geq c_H^*$ and $c_L \leq c_L^*$, there exists an SE. Suppose that $\lambda < \bar{\lambda}$. Then there exist c_H^* and c_L^* such that: (i) If $c_L \geq c_L^*$, there exists an NE; (ii) If either $c_H \leq c_H^*$, or $c_L \leq c_H^*$ and $c_H - (1-\lambda)c_L \leq c_H^* - (1-\lambda)(c_L^* - \pi^0)$, there exists an FE; (iii) If $c_H \geq c_H^*$, $c_L \leq c_L^*$ and $c_H - (1-\lambda)c_L \geq c_H^* - (1-\lambda)(c_L^* - \pi^0)$, there exists an SE. The proof of these two cases is relegated to Appendix C. To avoid redundant discussion, we will focus on the case where $\lambda \geq \bar{\lambda}$ in the following discussion.

We are especially interested in the conditions for an SE where firm H offers a normal product with a rate of return r^*. These conditions can be obtained by plugging $r_H = r^*$ into the proof of Proposition 4, so we state them here without an extra proof: Suppose that $\lambda \geq \bar{\lambda}$. Then, there exists an SE where firm H plays $(r^*, 0)$ if either $c_L \leq c_H^* \leq c_H$, or $c_L \geq c_H^*$ and $c_H \geq c_L^* - (1-\lambda)\pi^0$.

In sum, when both firms have low costs of committing financial fraud, they will offer fraudulent products. When their costs are high, both firms behave honestly and an NE arises. SE occurs between these two regions.

Corollary 1 shows that education also reduces a firm's incentive to commit financial fraud in a leader-follower model; this is consistent with Proposition 3.

Corollary 1. *c_H^* and c_L^* both increase with λ, and there exists λ_H^* such that when $\lambda \leq \lambda_H^*$, $c_H^* \leq 0$.*

It is worth noting that firm H may commit financial fraud in an SE. Although the market leader faces more severe punishment than the follower, there are still cases where the market leader commits financial fraud, while the follower behaves honestly. According to the proof of Proposition 4, this is the case when $c_H \leq c_L^* - (1-\lambda)\pi^0$, or $c_H - (1-\lambda)c_L \leq \lambda c_L^*$. Therefore, by comparing these two cutoffs with c^* in Proposition 1 for the case of monopoly, we can determine whether having firm L in the market changes firm H's incentive to commit financial fraud. Besides, we can also discuss whether firm L's presence affects investors' welfare. Here are the results:

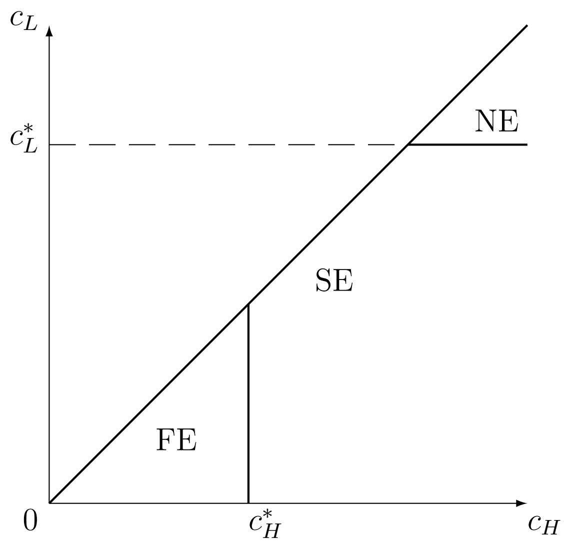

Figure 4: Graphical illustration of Proposition 4 ($\lambda \geq \bar{\lambda}$)

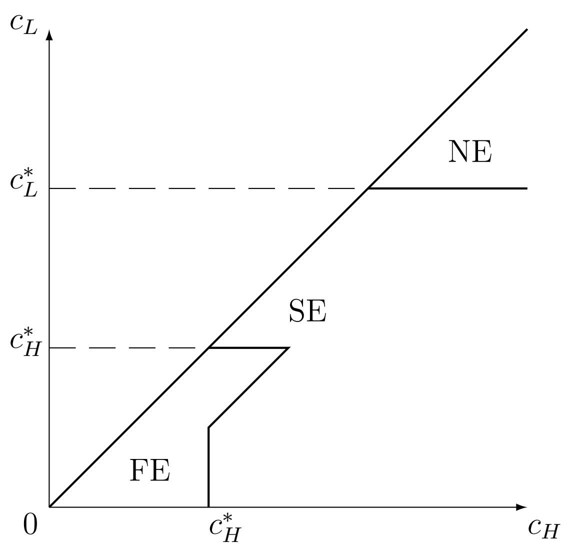

Figure 5: Graphical illustration of Proposition 4 ($\lambda < \bar{\lambda}$)

Proposition 5. *a). When $\lambda \geq \frac{\pi_s^1}{\pi_s^1+\pi^0}$, we have $c^* \leq c_L^* - (1-\lambda)\pi^0$; when $c_L \geq c_L^* + \pi_s^1 - \frac{\pi^0}{1-\lambda}$, we have $c^* \leq \lambda c_L^* + (1-\lambda)c_L$. That is, when either λ or c_L is sufficiently large, firm H has a stronger incentive to commit financial fraud in a leader-follower model than in a monopoly. Having firm L in the market exacerbates financial fraud.*

b). Suppose that firm H plays $(r^*, 0)$ in an SE. Then, $W_{FE}^c = W_F^m < W_{SE}^c < W_N^m < W_{NE}^c$. In other words, having firm L in the market will make investors worse off.

Intuitively, a monopolistic firm can sell its product to the whole market; however, in a leader-follower environment, firms earn zero profit in the NE. If any of them deviates to offer a fraudulent product, it immediately obtains a positive profit that is the same as that of a monopoly, as it can capture most of the naive investors. Therefore, selling a normal product becomes undesirable, while selling a fraudulent product turns to be attractive to the leader when a follower is present.[17]

Proposition 5 suggests that opening the monopoly market may harm investors: If the incumbent firm offers a normal product in a monopoly, while the entrant offers a fraudulent product and the market equilibrium is an SE or FE, then investors may be worse off after firm L's entry. Intuitively, having a follower drives each firm's profit in an NE down to zero, making it easier for firms to deviate and offer a fraudulent product. The welfare loss comes from naive investors who purchase normal products in a monopoly but get exploited by fraudulent products in an SE.

Our findings echo several theoretical and empirical results in the literature. Shleifer (2004) provides several examples in which unethical conduct emerges as a result of market competition. The key idea is that unethical conduct sometimes reduces costs or raises revenues, so firms may engage in censured behaviors as a response to intensified competition. This intuition is similar to that of Proposition 5, but we provide a formal model of this effect. Relatedly, Ru and Schoar (2016) find that less-sophisticated households are much more likely to be offered credit cards with back-loaded or hidden fees as a result of the screening strategies implemented by credit card companies, but this paper does not specifically focus on competition between firms. Agarwal et al. (2017) document that deregulation and competition increase the proportion of naive borrowers and, thus, may change the former unshrouded-price equilibrium into a shrouded-price equilibrium. Di Maggio et al. (2018) find that deregulation intensifies competition and increases the supply of more complex and risky mortgages. In Herweg and Rosato (2020), two firms compete in a retail market with naive buyers, but one of them is controlled by a manager contracting with its owner. The manager suffers from agency frictions and thus has a stronger incentive to exploit naive buyers. Herweg and Rosato (2020) also obtain market segregation results, but their results are driven by two firms facing different degrees of agency frictions.

[17] As we assume Bertrand competition, having more than two firms may drive the profit from selling normal products to zero. If we postulate another model of competition such as introducing product differentiation, most results can be preserved with multiple firms. In the present paper, we consider a two-firm Bertrand setting mainly for simplicity.

3.1 Information disclosure

Suppose that firms can costlessly disclose information to investors or unshroud (Gabaix and Laibson, 2006) about the fact that $x = 1$ is included in their choice sets when making offers to investors. Information disclosure changes some naive investors into sophisticated ones. We assume that the fraction of naive investors will change from λ to $\lambda - \delta$ ($\delta \in (0, \lambda)$) as long as one of the two firms decides to unshroud.

We restrict our attention to the combinations of c_L and c_H that lead to an SE (without information disclosure). The case of SE is a more prevalent scenario because most financial markets consist of both honest firms and fraudulent firms. Egan et al. (2019) find that, in markets of financial advisory, firms with clean records coexist with firms frequently engaging in misconduct. In the residential mortgage-backed securities market, there are firms intentionally provide buyers false information on asset characteristics, which leads to severe losses to buyers (Piskorski et al., 2015).

Firm L offering a fraudulent product has no incentive to unshroud. Firm H, which behaves honestly, may have an incentive to unshroud because doing so increases the proportion of sophisticated investors. As investors become aware of the possible financial fraud, they will stop purchasing the fraudulent product offered by firm L.

However, firm H does not always want to unshroud. In an SE, firm L and firm H both obtain some market power by offering differentiated products to two groups of investors. Both firms earn positive profits in this situation. However, reducing λ may change the market equilibrium into an NE. Recall that c_L^* is increasing in λ, so reducing λ has two effects: On the one hand, it raises the proportion of sophisticated investors, thereby making it more profitable to offer a normal product, and benefits firm H. On the other hand, as offering a normal product becomes attractive, it may induce firm L to deviate from committing financial fraud and provide a normal product to compete with firm H. As a consequence, when δ is small, firm H may have to increase its rate of return offered to investors and forgo some of its profits; when δ is large, information disclosure may change an SE into an NE and drive firm H's profit down to zero. Therefore, whether firm H is willing to unshroud depends on the aggregation of these two opposite driving forces.

In the proof of Proposition 4, we show that firm H's profit in an SE is bounded above by firm L's incentive constraint. Thus, firm H's profit will be increased only when this incentive constraint still holds after information disclosure, otherwise unshrouding makes firm H worse off. Therefore, firm H will unshroud only if

$$(\lambda - \delta)\pi_n^1 - c_L \geq \pi^0 \;\Leftrightarrow\; \delta \leq \lambda - \frac{c_L + \pi^0}{\pi_n^1} \equiv \delta^*.$$

Here, we assume that firm H will unshroud if it is indifferent and that the market equilibrium will not switch in the boundary cases. Proposition 6 formally states this result.

Proposition 6. *There exists δ^* such that in an SE with $r_H = r^*$, firm H will unshroud if and only if $\delta \leq \delta^*$.*

In other words, when financial fraud is present, an honest firm has some private incentive to educate investors such that some naive investors who would be attracted by the fraudulent product become aware of the danger and purchase the normal product offered by the honest firm. However, the incentive is limited: The honest firm does not want to reduce the proportion of naive investors to the extent that offering a fraudulent product becomes unprofitable, which intensifies the competition in the normal product market.[18] From Proposition 6, δ^* decreases with c_L, implying that firm H is more likely to shroud if firm L's cost advantage in exploiting naive investors is small.

Proposition 6 provides a new angle for the firm's incentive to shroud in a competitive market. In Gabaix and Laibson (2006) and Heidhues et al. (2017), naive consumers are exploited by firms through hidden add-ons. In Gabaix and Laibson (2006), a firm cannot convert its competitor's naive consumers by educating (or debiasing) them because these educated consumers can now benefit from products with shrouded prices. In Heidhues et al. (2017), Bertrand competition drives down the transparent part of the prices but not the add-on prices due to consumer naivety, so firms will shroud their add-on prices to avoid competition on the total prices. In our model, firm L has a cost advantage in exploiting naive investors and, thus, chooses not to compete with firm H for sophisticated investors. By shrouding, firm H restricts the profitability of offering a normal product and indirectly discourages firm L from competing with it. In contrast, shrouding directly increases the profitability of firms in Gabaix and Laibson (2006) and Heidhues et al. (2017).

In our model, firms cannot influence the proportions of naive and sophisticated investors but may design their products to target a certain type of investor. Firms cannot obfuscate investors, i.e., affect investors' awareness. Fraudulent products are more likely to appear in a leader-follower market only because having a competing firm makes the market for sophisticated investors less attractive.[19]

[18] If firms can control the proportion of investors receiving the disclosed information, i.e., δ becomes a choice variable, then firm H will choose $\delta = \delta^*$.

[19] If two firms move simultaneously, we can reach a similar result as Proposition 5. However, firm H will unshroud as it cannot affect firm L's decision. Proposition 6 does not hold, but the equilibrium segregation results preserve. A detailed discussion can be found in Appendix B.

3.2 Policy implications

The model brings forth several avenues for the analysis of regulations and policies. We consider three policy instruments: increasing legal punishment, implementing a public education program, and lowering the interest rate ceiling. Increasing legal punishment raises firms' costs of committing financial fraud, while public education programs reduce firms' profits from offering fraudulent products. In the absence of private information disclosure, they are clearly beneficial to investors. Lowering the interest rate ceiling reduces the profitability of fraudulent products and, thus, improves investors' welfare as long as it is not binding for normal products.

However, the welfare effects of these policies are ambiguous when competition and unshrouding are both present. All three policies may reduce firms' private incentive to disclose information. We focus on the SE case since it is most relevant to policy design.

Increasing legal punishment. Suppose that policymakers increase c_L to c'_L.[20] Such a policy can be interpreted as an increase in legal punishment, or auditing power, and thereby increases firm L's cost of committing financial fraud. As in the previous case, the direct effect of increasing c_L is to make financial fraud a less favorable choice for firm L, while the indirect effect is to make firm H less likely to disclose information about its choice set. Similarly, increasing c_L to c'_L will prevent firm H from unshrouding if and only if $\lambda - \frac{c'_L + \pi^0}{\pi^1_n} < \delta \leq \delta^*$.

If we further assume that $\lambda - \frac{c'_L + \pi^0}{\pi^1_n} \geq 0$, then firm H offers $r_H = r^*$ before and after the policy, and the policy does not transform the current SE into an NE. However, investors' welfare is decreased because firm H loses its incentive to unshroud. Our analysis can be summarized in Proposition 7.

Proposition 7. *Suppose that* $0 \leq \lambda - \frac{c'_L + \pi^0}{\pi^1_n} < \delta \leq \delta^*$. *Then, raising* c_L *to* c'_L *is not welfare-improving.*

The discussion on the effect of legal punishment can date back to the seminal paper by Becker (1968), which provides an economic approach to analyze the optimal level of punishment. In law and economics, there are also papers that emphasize the role of courts in protecting consumers with bounded rationality or asymmetric information (Korobkin, 2003; Becher, 2008), but usually they do not have a complete analysis with a full model and are loosely related to industrial organization.

Implementing a public education program. Suppose that policymakers also own a technology that can reduce λ to λ', which may stand for a public education program about

[20] There is no need to increase c_H because firm H already offers a normal product in the SE.

the possibility of firms committing financial fraud. Then, according to Proposition 6, when $0 \leq \lambda' - \frac{c'_L + \pi^0}{\pi_n^1} < \delta \leq \delta^*$, the public education program does not change the rates of return offered by firm L and firm H but crowds out firm H's incentive to disclose information, which is effective in naive investors of measure δ. Therefore, if the public education program is not as effective as firm H's unshrouding behavior, i.e., $\lambda - \lambda' \leq \delta$, investors' welfare will not increase due to the crowding-out effect. Proposition 8 characterizes the condition under which the crowding-out effect occurs.

Proposition 8. *Suppose that* $\lambda - \lambda' \leq \delta$ *and that* $0 \leq \lambda' - \frac{c'_L + \pi^0}{\pi_n^1} < \delta \leq \delta^*$. *Then, reducing* λ *to* λ' *is not welfare-improving.*

Gabaix and Laibson (2006) suggest that regulators can warn consumers to pay attention to shrouded costs, which is similar to the public education above. However, in Gabaix and Laibson (2006), educated consumers would exploit the firm with a shrouded price and would not deviate to the unshrouding firm. Thus, education is not preferred by the firm that behaves honestly. Li et al. (2016) study the effect of mandatory disclosure policy and find that it may discourage the firm's investment and harm consumers from an ex ante perspective. In their model, investment is costly, while in the present paper, firms have limited incentive to disclose even if doing so is costless.

Lowering the interest rate ceiling. Suppose that policymakers reduce $\bar{R}$ to $\bar{R}'$. Namely, a lower interest rate ceiling is imposed on the financial products sold in the market. Then, depending on the relationship between $\bar{R}'$ and r^*, there are two different cases.

If $\bar{R}' > r^*$ still holds, then the new interest rate ceiling's direct effect is to make the fraudulent product less attractive to naive investors, because firm L would advertise the highest possible rate of return for its product. Thus, financial fraud becomes less favorable to firm L, and fraudulent products are less likely to appear in the market. However, this policy also weakens firm H's incentive to unshroud because now firm L is more likely to offer a normal product and become a competitor of firm H. Mathematically, a decrease in $\bar{R}$ implies an increase in $\underline{\alpha}(\bar{R})$, which leads to decreases in π_n^1 and δ^*. In particular, lowering $\bar{R}$ to $\bar{R}'$ will change firm H's decision on information disclosure from unshrouding to shrouding if

$$\lambda - \frac{c_L + \pi^0}{p(R - \underline{r})[1 - F(\underline{\alpha}(\bar{R}'))]} < \delta \leq \delta^*, \tag{4}$$

where the left-hand side is the new cutoff that makes firm H indifferent between unshrouding and shrouding after the imposition of the policy.

We can also compute the welfare effect of lowering the interest rate ceiling. Suppose that

(4) is satisfied. Without this policy, firm H will disclose, and thus, investors' welfare is

$$(\lambda-\delta)\int_{\underline{\alpha}(\bar{R})}^{+\infty}[p\underline{r}^{\alpha}-I^{\alpha}]dF(\alpha)+(1-\lambda+\delta)\int_{\underline{\alpha}(r^*)}^{+\infty}[p(r^*)^{\alpha}-I^{\alpha}]dF(\alpha).$$

If this policy is implemented, firm H will shroud, and investors' welfare is

$$\lambda\int_{\underline{\alpha}(\bar{R}')}^{+\infty}[p\underline{r}^{\alpha}-I^{\alpha}]dF(\alpha)+(1-\lambda)\int_{\underline{\alpha}(r^*)}^{+\infty}[p(r^*)^{\alpha}-I^{\alpha}]dF(\alpha).$$

Therefore, the welfare effect of this policy is negative if

$$-\lambda\int_{\underline{\alpha}(\bar{R})}^{\underline{\alpha}(\bar{R}')}[p\underline{r}^{\alpha}-I^{\alpha}]dF(\alpha)<\delta\Big\{\int_{\underline{\alpha}(r^*)}^{+\infty}[p(r^*)^{\alpha}-I^{\alpha}]dF(\alpha)-\int_{\underline{\alpha}(\bar{R}')}^{+\infty}[p\underline{r}^{\alpha}-I^{\alpha}]dF(\alpha)\Big\}. \quad (5)$$

Proposition 9 summarizes our discussion.

Proposition 9. *Suppose that $\bar{R}'>r^*$ and that δ satisfies (4) and (5). Then, reducing $\bar{R}$ to $\bar{R}'$ is not welfare-improving.*

If $\bar{R}'\leq r^*$, the firm offering a normal product will find it optimal to choose $\bar{R}'$, since its profit function is nondecreasing in $[0,r^*]$. Similarly, the firm offering a fraudulent product will also choose $\bar{R}'$ to attract naive investors. No firm can deviate from offering $\bar{R}'$, but whether they will commit financial fraud depends on their costs. This policy is also distortionary and may be harmful to investors, since sophisticated investors cannot obtain a return of r^* from the honest firm.

There are papers that empirically study the effect of interest rate ceilings. Benmelech and Moskowitz (2010) show that a binding interest rate ceiling reduces credit and economic activity, especially for small firms. A tight interest rate ceiling is in fact a distortionary policy that restricts the freedom of contract.

In summary, the net effect of policy intervention is ambiguous when firms can disclose information to investors. If public policies cannot change an SE into an NE, or make the high-cost firm increase its rate of return offered to investors, the high-cost firm's incentive to unshroud may be weakened due to the fear of inducing competitors. Based on similar logic, a public education program may crowd out firms' unshrouding behaviors. In the behavioral industrial organization literature, there are other examples in which firms will strategically respond to policy interventions and jeopardize the effectiveness of regulation. Policies can harm consumers because firms respond by changing the attributes of their products. For instance, Murooka and Schwarz (2018); Johnen (2019) show that choice-enhancing policies that make it easier for consumers to switch can decrease consumer and social welfare if the firms adjust their pricing strategies in response. In this paper, we characterize a new mechanism that may lead to perverse welfare effects of policy interventions. As we separate the firm's product choice and information disclosure strategy, firms respond to policies by changing their decisions on information disclosure despite that they do not change their products.

4 Conclusion

Widespread financial fraud has emerged as a pressing problem in many countries. Investors are attracted by unrealistically high returns because some of them may not have a proper awareness of the underlying high risks and the possibility of financial fraud. In this paper, we model how boundedly rational investors get exploited by firms providing fraudulent financial products, and show that reducing the proportion of naive investors not only directly helps these investors but also attenuates or even eliminates the firms' incentive to commit financial fraud. Therefore, educating investors to be aware that unrealistically high returns come with high risks as a result of financial fraud is essential for developing a healthy financial market.

Our model offers several implications on competition and public policies. We find that there exists a separating equilibrium in which both firms earn positive profits. Having naive investors allows the two firms to offer differentiated products targeting two types of investors. In this equilibrium, the honest firm offering a normal product to sophisticated investors may have an incentive to unshroud the possibility of financial fraud because doing so increases its market share. However, if the proportion of naive investors becomes too small, the previously dishonest firm will turn to compete for sophisticated investors. Because of the second force, the honest firm may be reluctant to take the welfare-improving action of disclosing information. Lowering the interest rate ceiling, increasing legal punishment, and implementing a public education program may all discourage the honest firm from unshrouding. Therefore, in protecting boundedly rational consumers, policymakers should devote more attention to the key mechanisms behind nudges to avoid undesirable outcomes.

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Appendix

A Firm Entry

In Section 3, firm L's entry is taken as given. Now we relax this assumption and study firm L's entry decision when it has to incur a positive fixed cost $k > 0$ to enter the market.

Proposition 10. *Suppose that $\lambda \geq \bar{\lambda}$. Consider the following three conditions: (i) $c_L > c_L^* - k$; (ii) $c_H < c^* + \lambda\pi^0$; (iii) $c_H - (1-\lambda)c_L \leq c^* + \pi^0 - (1-\lambda)c_L^*$.*

a). When $k \leq (1-\lambda)\pi^0$, firm L's entry is deterred if (i) holds. In this case, firm H offers a normal product.

b). When $(1-\lambda)\pi^0 < k \leq \pi^0$, firm L's entry is deterred if $c_L > c_H^* + (1-\lambda)\pi^0 - k$, and one of (i), (ii) and (iii) holds. In this case, firm H offers a normal product if $c_L > c_L^* - k$ and $c_H > c^* + \pi^0 - k$; otherwise it offers a fraudulent product.

c). When $k > \pi^0$, firm L's entry is deterred if $c_L > c_H^* + (1-\lambda)\pi^0 - k$, and (i) or (ii) holds. In this case, firm H offers a normal product if $c_L > c_L^* - k$ and $c_H > c^*$; otherwise it offers a fraudulent product.

d). When $c_L \leq c_H^* + (1-\lambda)\pi^0 - k$ and $c_H \leq c_H^*$, firm L's entry is accommodated, and there exists an FE.

e). Otherwise, firm L's entry is accommodated, and there exists an SE.

Figure A.6 gives a graphical illustration of Proposition 10 with an intermediate entry cost, i.e., $(1-\lambda)\pi^0 < k \leq \pi^0$.

We want to make several remarks regarding Proposition 10. First, the effect of the entry cost on entry deterrence is monotone; a higher entry cost results in less entry. To see this, note that condition (iii) implies either $c_H < c^* + \lambda\pi^0$ or $c_L > c_L^* - \pi^0$, the latter inequality is sufficient for $c_L > c_L^* - k$ when $k > \pi^0$.

However, a higher entry cost does not imply less financial fraud. It is worth mentioning that there is no NE when the entry cost is positive, since both firms earn zero profit in an NE. Therefore, naive investors are free from exploitation only when firm H offers a normal product to deter entry. This is the case when c_H and c_L are both sufficiently large. Nevertheless, there is no monotonic relationship between k and the cutoff for c_H, suggesting that increasing the entry barrier for low cost firms may not guarantee a welfare improvement for naive investors.

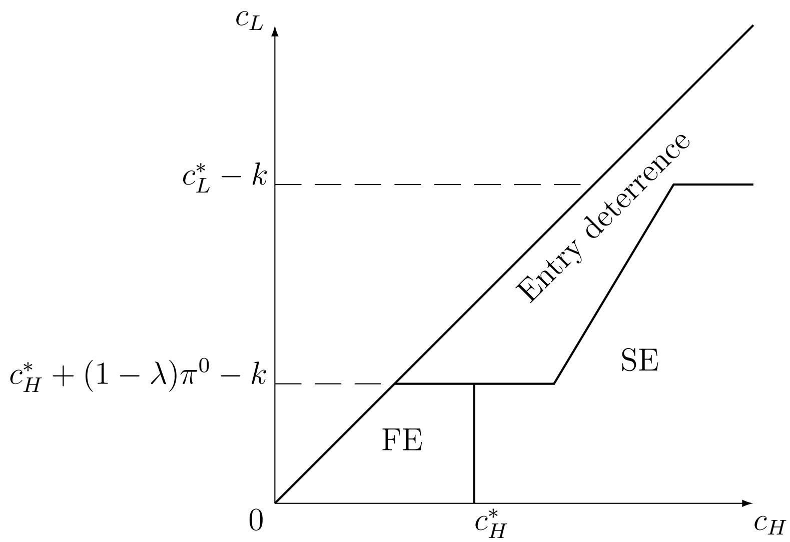

Figure A.6: Graphical illustration of Proposition 10 ($(1-\lambda)\pi^0 < k \leq \pi^0$)

We also find that, whether the incumbent provides a normal product to deter entry relies heavily on the new entrant's entry cost k as well as its cost of committing financial fraud c_L. These two costs are complimentary because condition (i) is essentially equivalent to $c_L + k > c_L^*$.

B Simultaneous Moves

In Section 3, we consider a leader-follower game and assume that two firms move sequentially. Now we study a variation of our model in which two firms move simultaneously. It can be shown that the market segmentation results preserve in this alternative setting.

Normal-product equilibrium (NE). In an NE, both firms offer normal products ($x_H = x_L = 0$). Competition drives returns up to $r_H = r_L = R$, and both firms earn zero profit. If firm j deviates to choosing $x_j = 1$, the most profitable strategy is to set $r_j = \bar{R}$. Naive investors will purchase only from firm j, and sophisticated investors will purchase only from the other firm, denoted firm k ($k \neq j$). Therefore, by choosing $x_j = 1$, firm j's profit is $\lambda\pi_n^1 - c_j$. Hence, $x_H = x_L = 0$ constitutes an equilibrium if,

$$\lambda\pi_n^1 - c_j \leq 0 \ \Leftrightarrow \ c_j \geq \lambda\pi_n^1 \equiv c_L^* \text{ for } j = L, H.$$

Because $c_L \leq c_H$, an NE with $r_H = r_L = R$ arises when $c_L \geq c_L^*$.

Fraudulent-product equilibrium (FE). In an FE, both firms commit financial fraud ($x_H = x_L = 1$). They will propose $r_H = r_L = \bar{R}$ to attract as many naive investors as possible, so each firm earns a profit $\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_j$.

If firm j deviates to $x_j = 0$, then the most profitable rate of return is $r_j = r^*$, which yields a profit $(1-\lambda)\pi^0$. $x_H = x_L = 1$ constitutes an equilibrium if

$$\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_j \geq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_j \leq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - (1-\lambda)\pi^0 \equiv c_H^*, \text{ for } j = L, H.$$

Because $c_L \leq c_H$, an FE with $r_H = r_L = \bar{R}$ arises when $c_H \leq c_H^*$. Note that $c_H^* \geq 0$ requires that

$$\lambda \leq \frac{2\pi^0 - \pi_s^1}{2\pi^0 - \pi_s^1 + \pi_n^1} \equiv \lambda_H^*.$$

Hence, similar to our discussion in Proposition 3, when the proportion of naive investors is small enough, c_H^* becomes negative, implying that there is no FE.

Separating equilibrium (SE). Suppose that, in an SE, firm j provides a normal product while firm k commits financial fraud, i.e. $x_j = 0$, $x_k = 1$.

In an SE, firm j's equilibrium strategy is $(r^*, 0)$, and firm k's equilibrium strategy is $(\bar{R}, 1)$. For firm j playing $x_j = 0$, the most profitable deviation from offering a normal product is to play $(\bar{R}, 1)$. Therefore, its incentive constraint is

$$(1-\lambda)\pi^0 \geq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_j \;\Leftrightarrow\; c_j \geq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - (1-\lambda)\pi^0 = c_H^*.$$

For firm k playing $x_k = 1$, its most profitable deviation is to play $(r^* + \varepsilon, 0)$, i.e., to offer a normal product with $r_k = r^* + \varepsilon$, where ε is a small positive number. Upon deviating, firm k is able to capture the whole market and earn a profit arbitrarily close to π^0. Therefore, firm k's incentive constraint is

$$\lambda\pi_n^1 - c_k \geq \pi^0 \;\Leftrightarrow\; c_k \leq \lambda\pi_n^1 - \pi^0 = c_L^* - \pi^0.$$

Hence, an SE can be supported if $c_j \geq c_H^*$ and $c_k \leq c_L^* - \pi^0$. These two conditions are equivalent to $c_H \geq c_H^*$ and $c_L \leq c_L^* - \pi^0$.

Proposition 11 summarizes our results when two firms move simultaneously.

Proposition 11. *There exist c_H^* and c_L^* such that:*

a). When $c_L \geq c_L^*$, there exists an NE in which both firms play $(R, 0)$.

b). When $c_H \leq c_H^*$, there exists an FE in which both firms play $(\bar{R}, 1)$.

c). When $c_H \geq c_H^*$ and $c_L \leq c_L^* - \pi^0$, there exists an SE in which firm H plays $(r^*, 0)$, and firm L plays $(\bar{R}, 1)$.

Therefore, our market segregation results preserve in this simultaneous-move setting. However, when there is information disclosure, firm H will disclose irrespective of firm L's decision.

C Proofs

C.1 Proof of Proposition 4

Normal-product equilibrium (NE). In an NE, both firms offer normal products ($x_H = x_L = 0$). Suppose that firm H chooses $r_H < R$, then firm L's best response would be choosing r_L slightly higher than r_H to capture the whole market. Therefore, $r_H < R$ cannot be played in an NE. Given that firm H chooses $r_H = R$ in the equilibrium, firm L's equilibrium strategy must be $r_L = R$, and both firms' equilibrium profit is zero.

If firm L deviates to choosing $x_L = 1$, the most profitable strategy is setting $r_L = \bar{R}$ to attract as many naive investors as possible. Upon such deviation, firm L's profit is $\lambda\pi_n^1 - c_L$. Thus, firm L's incentive constraint is

$$\lambda\pi_n^1 - c_L \leq 0 \;\Leftrightarrow\; c_L \geq \lambda\pi_n^1 \equiv c_L^*.$$

Now suppose $c_L \geq c_L^*$. If firm H deviates to choosing $x_H = 1$, it will also set $r_H = \bar{R}$. Observing firm H's deviation, firm L will stick to $x_L = 0$. Firm H's profit upon deviation is $\lambda\pi_n^1 - c_H$, which is nonpositive. Hence, an NE exists if and only if $c_L \geq c_L^*$.

Fraudulent-product equilibrium (FE). In an FE, both firms commit financial fraud ($x_H = x_L = 1$). They will propose $r_H = r_L = \bar{R}$ to attract as many naive investors as possible, so each firm earns a profit $\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_j$.

If firm L deviates to choosing $x_L = 0$, the most profitable strategy is setting $r_L = r^*$ to maximize its profit from selling a normal product to sophisticated investors. Upon such deviation, firm L's profit is $(1-\lambda)\pi^0$. Thus, firm L's incentive constraint is

$$\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_L \geq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_L \leq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - (1-\lambda)\pi^0 \equiv c_H^*.$$

If firm H deviates to choosing $x_H = 0$ and $r_H < R$, it can make a positive profit only when firm L does not follow its deviation and sticks to $x_L = 1$. Put differently, if firm L also deviates and plays $x_L = 0$, it will capture the whole by setting r_L slightly above r_H. Firm H only gets zero profit from deviation. This is the case if

$$\lambda\pi_n^1 - c_L \leq p(R - r_H)[1 - F(\underline{\alpha}(r_H))]. \tag{6}$$

Moreover, even if (6) is violated, and firm H can make a positive profit upon deviation, it is also possible that such profit is less than firm H's equilibrium profit. In other words, firm H's incentive constraint is

$$\frac{1}{2}[\lambda\pi_n^1 + (1 - \lambda)\pi_s^1] - c_H \geq (1 - \lambda)p(R - r_H)[1 - F(\underline{\alpha}(r_H))]. \tag{7}$$

Combining (6) and (7) gives us either $c_H \leq c_H^*$, or

$$c_H - (1 - \lambda)c_L \leq \frac{1}{2}[\lambda\pi_n^1 + (1 - \lambda)\pi_s^1] - (1 - \lambda)\lambda\pi_n^1 = c_H^* - (1 - \lambda)(c_L^* - \pi^0).$$

Hence, an FE exists if and only if either $c_H \leq c_H^*$, or $c_L \leq c_H^*$ and $c_H - (1 - \lambda)c_L \leq c_H^* - (1 - \lambda)(c_L^* - \pi^0)$.

Separating equilibrium (SE). Suppose that, in an SE, firm H provides a normal product with $r_H < R$, while firm L commits financial fraud with $r_L = \bar{R}$. Firm H's equilibrium profit is $(1 - \lambda)p(R - r_H)[1 - F(\underline{\alpha}(r_H))]$, and firm L's equilibrium profit is $\lambda\pi_n^1 - c_L$.

If firm L deviates to choosing $x_L = 0$, its most profitable strategy is setting r_L slightly above r_H to capture the whole market. Thus, firm L's incentive constraint is

$$\lambda\pi_n^1 - c_L \geq p(R - r_H)[1 - F(\underline{\alpha}(r_H))].$$

A necessary condition is $c_L \leq c_L^*$. Note that, if the right-hand side is strictly less than π^0, this constraint must be binding, otherwise firm H will change its r_H to increase its profit. In other words,

$$p(R - r_H)[1 - F(\underline{\alpha}(r_H))] = \min\{\lambda\pi_n^1 - c_L, \pi^0\}.$$

If firm H deviates to choosing $x_H = 1$ and $r_H = \bar{R}$, firm L will either sticks to $x_L = 1$ or switches to $x_L = 0$. The former case happens when

$$\frac{1}{2}[\lambda\pi_n^1 + (1 - \lambda)\pi_s^1] - c_L \geq (1 - \lambda)\pi^0 \;\Leftrightarrow\; c_L \leq c_H^*.$$

In this case, firm H will earn $\frac{1}{2}[\lambda\pi_n^1 + (1 - \lambda)\pi_s^1] - c_H$ upon deviation. Its incentive constraint is

$$(1-\lambda)p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_H,$$

which implies $c_H \geq c_H^*$ and $c_H - (1-\lambda)c_L \geq c_H^* - (1-\lambda)(c_L^* - \pi^0)$.

The latter case happens when

$$\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_L \leq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_L \geq c_H^*.$$

In this case, firm H will earn $\lambda\pi_n^1 - c_H$ upon deviation. Its incentive constraint is

$$(1-\lambda)p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \lambda\pi_n^1 - c_H,$$

which implies $c_H \geq c_L^* - (1-\lambda)\pi^0$ and $c_H - (1-\lambda)c_L \geq \lambda c_L^*$.

Consider an alternative case where firm H commits financial fraud with $r_H = \bar{R}$, and firm L provides a normal product with $r_L = r^*$ in an SE. By a similar discussion, firm L's incentive constraint is

$$(1-\lambda)\pi^0 \geq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_L \;\Leftrightarrow\; c_L \geq c_H^*.$$

If firm H deviates to choosing $x_H = 0$, firm L will stick to $x_L = 0$ when

$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \lambda\pi_n^1 - c_L. \tag{8}$$

In this case, firm H's profit upon deviation is zero. Alternatively, firm L will switch to $x_L = 1$ when (8) is violated. In this case, firm H's incentive constraint is

$$\lambda\pi_n^1 - c_H \geq (1-\lambda)p(R-r_H)[1-F(\underline{\alpha}(r_H))]. \tag{9}$$

Combining (8) and (9) gives us either $c_H \leq c_L^* - (1-\lambda)\pi^0$, or $c_H - (1-\lambda)c_L \leq \lambda c_L^*$.

C.2 Proof of Proposition 5

Proving part (a) of the proposition is straightforward, so we only need to verify part (b).

In an NE, investors with $\alpha \geq \underline{\alpha}(R)$ purchase both firms' normal products with equal probability. Investors' welfare is

$$W_{NE}^c = \int_{\underline{\alpha}(R)}^{+\infty} [pR^\alpha - I^\alpha] dF(\alpha).$$

Compared to the welfare with a monopoly offering a normal product given in (2), we find that competition improves welfare, i.e., $W^c_{NE} > W^m_N$, because the rate of return for the normal product increases from r^* to R.

In an FE, sophisticated investors with $\alpha \geq \underline{\alpha}(\underline{r})$ and naive investors with $\alpha \geq \underline{\alpha}(\bar{R})$ purchase both firms' fraudulent products with equal probability. Investors' welfare is

$$W^c_{FE} = W^m_F = \lambda \int_{\underline{\alpha}(\bar{R})}^{+\infty} [p\underline{r}^\alpha - I^\alpha] dF(\alpha) + (1-\lambda) \int_{\underline{\alpha}(\underline{r})}^{+\infty} [p\underline{r}^\alpha - I^\alpha] dF(\alpha),$$

where W^m_F is given in (3). Hence, competition does not improve welfare if firms offer fraudulent products.

In an SE, sophisticated investors with $\alpha \geq \underline{\alpha}(r^*)$ purchase firm H's normal product, and naive investors with $\alpha \geq \underline{\alpha}(\bar{R})$ purchase firm L's fraudulent product. Investors' welfare is

$$W^c_{SE} = \lambda \int_{\underline{\alpha}(\bar{R})}^{+\infty} [p\underline{r}^\alpha - I^\alpha] dF(\alpha) + (1-\lambda) \int_{\underline{\alpha}(r^*)}^{+\infty} [p(r^*)^\alpha - I^\alpha] dF(\alpha),$$

which lies in between welfare measures under NE and FE.

C.3 Proof of Proposition 10

We consider the following cases separately:

- Case 1: Firm H offers a normal product; firm L enters and offers a fraudulent product;
- Case 2: Firm H offers a normal product; firm L does not enter;
- Case 3: Firm H offers a fraudulent product; firm L enters and offers a normal product;
- Case 4: Firm H offers a fraudulent product; firm L enters and offers a fraudulent product;
- Case 5: Firm H offers a fraudulent product; firm L does not enter.

Case 1. Firm L's incentive constraints are

$$k \leq \lambda \pi^1_n - c_L,$$
$$\lambda \pi^1_n - c_L \geq p(R - r_H)[1 - F(\underline{\alpha}(r_H))].$$

If firm H deviates to offering a fraudulent product, the first inequality suggests that firm L will at least enter the market and offer a normal product at a rate of return r^*. Hence, firm

H's incentive constraint is

$$(1-\lambda)p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_H \qquad \text{if } c_L \leq c_H^*,$$
$$(1-\lambda)p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \lambda\pi_n^1 - c_H \qquad \text{if } c_L \geq c_H^*,$$

which implies

$$c_H \geq c_H^*,\ c_H - (1-\lambda)c_L \geq c_H^* - (1-\lambda)(c_L^* - \pi^0) \qquad \text{if } c_L \leq c_H^*,$$
$$c_H \geq c_L^* - (1-\lambda)\pi^0,\ c_H - (1-\lambda)c_L \geq \lambda c_L^* \qquad \text{if } c_L \geq c_H^*.$$

Note that $c_H - (1-\lambda)c_L \geq c_H^* - (1-\lambda)(c_L^* - \pi^0)$ is implied by $c_H \geq c_H^*$ and $c_L \leq c_H^*$.

Case 2. Firm L's incentive constraints are

$$k > \max\{p(R-r_H)[1-F(\underline{\alpha}(r_H))], \lambda\pi_n^1 - c_L\}.$$

If $k \geq \pi^0$, firm H will choose $r_H = r^*$, and it can be verified that when firm H deviates to offering a fraudulent product, firm L will not enter. Hence, firm H's incentive constraint is $c_H \geq c^*$. If $k < \pi^0$, firm H will choose r_H according to

$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] = k.$$

When firm H deviates to offering a fraudulent product, firm L will enter and offer a normal product at a rate of return r^* if $k \leq (1-\lambda)\pi^0$; otherwise firm L will not enter. Therefore, Firm H's incentive constraint is

$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \lambda\pi_n^1 - c_H, \qquad \text{if } k \leq (1-\lambda)\pi^0,$$
$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] \geq \lambda\pi_n^1 + (1-\lambda)\pi_s^1 - c_H \qquad \text{if } k > (1-\lambda)\pi^0.$$

In the first inequality, the left-hand side equals k, so it can be replaced by firm L's incentive constraint. In sum, an equilibrium exists if $k > c^* - c_L$, and

$$k \leq (1-\lambda)\pi^0,$$
$$(1-\lambda)\pi^0 < k \leq \pi^0,\ \text{and } c_H \geq c^* + \pi^0 - k,$$
$$k > \pi^0,\ \text{and } c_H \geq c^*.$$

Case 3. Firm L's incentive constraints are

$$k \leq (1-\lambda)\pi^0,$$
$$(1-\lambda)\pi^0 \geq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_L \ \Leftrightarrow \ c_L \geq c_H^*.$$

If firm H deviates to offering a normal product, firm L's response is determined by the relationship between $\lambda\pi_n^1 - c_L$, k and π^0. If $\lambda\pi_n^1 - c_L < k$, firm H's most profitable deviation is to choose r_H according to

$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] = k$$

to deter firm L's entry. If $k \leq \lambda\pi_n^1 - c_L < \pi^0$, firm H's most profitable deviation is to choose r_H according to

$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] = \lambda\pi_n^1 - c_L$$

to accommodate firm L's entry. If $\lambda\pi_n^1 - c_L \geq \pi^0$, firm H's most profitable deviation is to choose $r_H = r^*$. Hence, firm H's incentive constraint is

$$\begin{aligned} &\lambda\pi_n^1 - c_H \geq k && \text{if } \lambda\pi_n^1 - c_L < k, \\ &\lambda\pi_n^1 - c_H \geq (1-\lambda)(\lambda\pi_n^1 - c_L) && \text{if } k \leq \lambda\pi_n^1 - c_L < \pi^0, \\ &\lambda\pi_n^1 - c_H \geq (1-\lambda)\pi^0 && \text{if } \lambda\pi_n^1 - c_L \geq \pi^0. \end{aligned}$$

Note that the first case is degenerate due to $c_H \geq c_L$. The second case can be simplified as $c_H - (1-\lambda)c_L \leq \lambda c_L^*$. The third case can be simplified as $c_H \leq c_L^* - (1-\lambda)\pi^0$.

Case 4. Similar to Case 3, we know that firm L's incentive constraints are

$$k \leq \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_L,$$
$$c_L \leq c_H^*.$$

If firm H deviates to offering a normal product, firm L's response is determined by the relationship between $\lambda\pi_n^1 - c_L$ and π^0. If $\lambda\pi_n^1 - c_L < \pi^0$, firm H's most profitable deviation is to choose r_H according to

$$p(R-r_H)[1-F(\underline{\alpha}(r_H))] = \lambda\pi_n^1 - c_L$$

to accommodate firm L's entry. If $\lambda\pi_n^1 - c_L \geq \pi^0$, firm H's most profitable deviation is to choose $r_H = r^*$. Hence, firm H's incentive constraint is

$$\begin{aligned} &\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_H \geq (1-\lambda)(\lambda\pi_n^1 - c_L) && \text{if } \lambda\pi_n^1 - c_L < \pi^0, \\ &\frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_H \geq (1-\lambda)\pi^0 && \text{if } \lambda\pi_n^1 - c_L \geq \pi^0. \end{aligned}$$

Note that $\lambda\pi_n^1 - c_L < \pi^0$ contradicts $c_L \leq c_H^*$. Only the second case is relevant and can be simplified as $c_H \leq c_H^*$.

Case 5. Similar to Case 2, firm L's incentive constraints are

$$k > \max\{(1-\lambda)\pi^0, \frac{1}{2}[\lambda\pi_n^1 + (1-\lambda)\pi_s^1] - c_L\}.$$

If firm H deviates to offering a normal product, firm L's response is determined by the relationship between $\lambda\pi_n^1 - c_L$, k and π^0. If $\lambda\pi_n^1 - c_L < k < \pi^0$, firm H's most profitable deviation is to choose r_H according to

$$p(R - r_H)[1 - F(\underline{\alpha}(r_H))] = k$$

to deter firm L's entry. If $k \leq \lambda\pi_n^1 - c_L < \pi^0$, firm H's most profitable deviation is to choose r_H according to

$$p(R - r_H)[1 - F(\underline{\alpha}(r_H))] = \lambda\pi_n^1 - c_L$$

to accommodate firm L's entry. Otherwise, firm H's most profitable deviation is to choose $r_H = r^*$. Hence, firm H's incentive constraint is

$$\begin{aligned} &\lambda\pi_n^1 + (1-\lambda)\pi_s^1 - c_H \geq k && \text{if } \lambda\pi_n^1 - c_L < k < \pi^0, \\ &\lambda\pi_n^1 + (1-\lambda)\pi_s^1 - c_H \geq (1-\lambda)(\lambda\pi_n^1 - c_L) && \text{if } k \leq \lambda\pi_n^1 - c_L < \pi^0, \\ &\lambda\pi_n^1 + (1-\lambda)\pi_s^1 - c_H \geq \pi^0 && \text{if } k > \max\{\lambda\pi_n^1 - c_L, \pi^0\}, \\ &\lambda\pi_n^1 + (1-\lambda)\pi_s^1 - c_H \geq (1-\lambda)\pi^0 && \text{if } k \leq \pi^0 \leq \lambda\pi_n^1 - c_L, \text{or } \pi^0 \leq k \leq \lambda\pi_n^1 - c_L. \end{aligned}$$

Summarizing all the five cases, we have the results stated in the proposition.