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Attraction Versus Persuasion

Pak Hung Au and Mark Whitmeyer

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1 Introduction

In many markets, consumers are unaware of essential aspects of the products available and can only discover and learn about them by interacting with the items in some way. These interactions are often costly for consumers: many goods require in-person encounters in order for important information to be transmitted, and such visits are not free. Indeed, the feel of a rug or a car, the fit of clothes or shoes, the sound of an instrument or speakers, the smell of perfume or wooden furniture, or the taste of a food or beverage are all attributes fundamental to their respective products that must be observed by the consumer, herself, in the flesh. While in-person inspections are not necessary for products like online newspapers, music albums, (e-)books, and software, consumers still need to expend costly time and effort to try free samples or conduct their own research so as to discover pertinent information.

Firms often have a great deal of control as to how much information about their products consumers' inspections will bring. Car sellers choose whether to offer test drives, software companies decide the length of free trial periods and the set of specific functions to include in promotional versions, newspapers limit how many free articles consumers can access, and book vendors specify how many and which pages consumers may sample for free. In choosing how much information to provide, a principal objective of each firm is clearly *persuasion*: each wants consumers to select its product over those of its competitors. On the other hand, because search is costly and time-consuming, consumers are selective about which products to investigate. Another principal objective is therefore *attraction*: each firm wants consumers to consider it (visit it and inspect its product) first.

In this paper, we investigate a series of fundamental questions: how are firms' information provision policies shaped by market competition? How do search frictions affect the intensity of competition? Would otherwise identical firms adopt common or idiosyncratic information policies? To answer these, we study a single-product oligopoly setting in which several firms compete by designing how much information a representative consumer obtains about their respective products through her (costly) inspections. We abstract away from price competition and focus on the information provision problem of the firms.[1]

[1]This is justified if the competing firms are dealers in some product the price of which is set by a central office.

In our model, each *ex-ante* identical firm has a product of uncertain quality, which is either high or low.[2] The quality distribution is independent and identically distributed across firms, with the probability of high quality denoted by μ. Each firm has complete freedom over how much information about its product's quality a consumer's visit reveals: each firm simultaneously chooses and commits *ex-ante* to a signal or experiment, the outcome of which is revealed to the consumer only upon visiting that firm. These chosen signals are publicly posted and are observed by the consumer prior to commencement of her search. The consumer needs at most one product.

Knowing only the collection of signals posted but not their realizations, the consumer decides which firms to visit and in what order, at a search cost of $c > 0$ per visit. As is standard in the consumer search literature, we assume that the consumer must visit a firm before she can buy from it and that recall is free: having visited a firm, the consumer may always return to select that firm's product. Consequently, at each stage in her sequential search, the consumer has two decisions to make: whether to stop (by selecting one product or the outside option) or continue, and whom to visit next if she continues.

With the collection of signal distributions fixed, the optimal search strategy, characterized by Weitzman (1979), takes a simple form. Given its choice of signal, each firm is assigned a reservation value characteristic to that firm. The consumer inspects the firms' products in descending order of their reservation values. If, at any time, the consumer finds a product that has a posterior expected quality exceeding the highest reservation value of the remaining firms, the consumer stops her search and selects the best product discovered thus far.[3]

As hinted above, there is an important tension present in the model, which drives our results. Namely, when choosing its signal, a firm trades off between *persuasion* and *attraction*. Conditional on the consumer visiting, a firm wishes to maximize the chance that it is selected. To this end, all beliefs above the firm's reservation value are equivalent, since all lead the consumer to stop and select that firm. Thus, the persuasion incentive encourages pooling of beliefs above the

Such firms can control the level of information that they provide but not the price.

[2]Alternatively, this quality can be interpreted as the consumer's match value with the product. We use quality throughout in order to easily differentiate between it and the reservation value assigned to the firm in the consumer's search problem.

[3]Our model extends Weitzman (1979) by endogenizing the collection of prize distributions from which the consumer samples.

stopping threshold, as firms try to maximize the conversion rate from visits to purchases. On the other hand, the reservation value rewards informativeness–the more information a firm provides, the higher its reservation value and the earlier it is visited in the consumer's search. *Ceteris paribus*, the earlier a firm is visited, the better for the firm, since ranking lower in the search order implies a greater chance that the consumer stops elsewhere before visiting it. In contrast to the persuasion incentive, the attraction incentive encourages the spreading of beliefs, in order to entice the consumer to visit. While the attraction motive encourages more informative signals, the persuasion motive calls for less. A firm's optimal signal is determined by the interplay of these two forces.

We find that when the average quality, μ, of the products is sufficiently high, the attraction motive dominates, and the unique symmetric equilibrium is one in which each firm chooses a fully informative signal. There is no profitable deviation from full information, as any other signal would ensure that the deviating firm be visited last, a rare event due to the surfeit of high quality products in the market. Conversely, if μ is not high, we find that there are no symmetric equilibria in pure strategies. Because the average quality is low, the persuasion motive is more important. A firm can deviate profitably from providing full information by being uninformative: even though it will be visited last–indeed, it will only be visited if every other firm is low quality–it will be selected for certain if visited. Nevertheless, the attraction incentive still remains and precludes the existence of any other symmetric pure strategy equilibrium, as firms can always deviate profitably by providing slightly more information and moving to the top of the consumer's search order.

In order to characterize the unique symmetric equilibrium outside of the high average quality case, we use recent results from Armstrong (2017) and Choi et al. (2018), who show how the sequential problem of Weitzman (1979) can be reformulated as a static discrete choice problem in which the consumer selects the firm with the highest realized *effective* value. Adapting the concavification method of Kamenica and Gentzkow (2011) to this setting, we establish that there is a unique symmetric equilibrium distribution over effective values, which necessarily begets a payoff function for each firm that is linear in the firm's realized effective value. Importantly, this distribution requires firms to randomize over their choices of signals, i.e., our model generates *information dispersion*.[4] In equilibrium, each firm mixes over a continuum of information levels, with no atoms except possibly on the fully informative signal, which arises when μ is not too low.

Our prediction of information dispersion is novel and suggests potentially profitable avenues for empirical study. For instance, in the market for antivirus/security software, all of the major companies offer potential consumers free trials, but differ in the length of these previews: 7 days for AVG and Avast, 14 days for MalwareBytes, and 30 days for Norton, McAfee and Kaspersky. Similarly, there is variation in the length of free trial periods offered by music streaming services: 1 month for Google Play Music and Tidal, 2 months for Pandora, and 3 months for Spotify and Apple Music. The market for grammar checking software features not only different free-trial periods, but also different functionalities (including word limits and style settings) in firms' trial offers.

To highlight the significance of the attraction motive in driving our results, we also investigate two benchmarks in which only the persuasion incentive is present, albeit for different reasons. In Section 5.1, we explore the case in which the consumer is able to observe the signal realizations of all of the firms for free (corresponding to setting $c = 0$ in our main model). In the absence of the attraction motive, firms have less incentive to provide information; and in the unique symmetric equilibrium, firms do not provide full information. Consequently, our results imply that when it is important to be visited early (μ is high), the consumer can actually benefit from having a small positive search cost $c > 0$, which engenders the attraction incentive.

In Section 5.2, we consider another benchmark in which the consumer can observe a firm's choice of signal only after paying it a visit. When information is hidden in this manner, the attraction motive is completely absent, and we point out a dramatic "informational Diamond paradox." Namely, the only equilibrium outcome is that firms provide no useful information and the consumer does not actively search. Deviations to other signals cannot be observed in advance and so the consumer's search order is determined entirely by her conjectures and not the actual signals. The pooling incentive is all that remains, which eliminates any purported equilibria with

[4] Note that by information dispersion, we mean the informational analog of price dispersion. This refers to the variation in levels of information provision among identical firms in the same market. This is different in nature from that cited in Marquez (2002), which refers to the increase in fragmentation of the borrower-specific information held by lenders as the number of competing lenders increases.

active search. Again, there is no information dispersion as all firms provide the monopoly level of information.

For tractability, our main model assumes *ex-ante* homogeneous firms, so it is natural to focus on symmetric equilibria. For simplicity, we also assume the consumer's outside option is so low that it is irrelevant. In Section 6, we explore the consequences of relaxing these restrictions. In Section 6.1, we illustrate that asymmetric equilibria are possible in some parameter regions. In Section 6.2, we analyze the competition between two asymmetric firms. In Section 6.3, we show that the introduction of a non-negligible outside option leaves the qualitative features of the equilibrium intact, and this modification allows us to investigate comparative statics results on firm profit.

This section concludes with a brief discussion of related work. The model is set up in Section 2. Section 3 reports some preliminary observations and explains how the game under study can be reformulated into a more tractable one. Results on equilibrium existence, uniqueness and characterization are detailed in Section 4. Section 5 illustrates the major economic forces at work by considering two benchmark models. A number of extensions are considered in Section 6. Section 7 concludes with some discussion of our results. All proofs are left to the appendices, unless stated otherwise.

1.1 Related Work

There are still relatively few papers that explore information design in search settings. One such paper is Board and Lu (2018). They also consider a setting in which sellers compete by designing experiments, and buyers search sequentially to learn about their products. In contrast to our paper, the sellers' experiments are not publicly posted and hence do not direct the buyers' search. The tension between attraction and persuasion, vital to our model, is absent from their setup. In addition, they assume that the state of the world upon which the buyers' utilities depend is common; whereas in our model, the information produced by each firm provides the consumer with no information about any of the other firms.

Board and Lu (2018) show that under certain conditions, the monopoly (no active search) outcome is a unique equilibrium. The intuition behind their result is similar to our finding in the hidden information setup–there is a strong incentive for firms to pool information just above a consumer's stopping threshold. Whitmeyer (2020) enriches the hidden information setting of this paper by allowing firms to set prices as well, and finds that regardless of whether prices are hidden or posted, the no active search result persists. Whether prices are posted does, however, affect consumer welfare: posted prices beget pricing at marginal cost whereas hidden prices lead to monopoly pricing.

Dogan and Hu (2018) explore consumer-optimal information structures in the sequential (undirected) search framework of Wolinsky (1986). They find that consumer welfare is maximized by a signal that generates a (conditional) unit-elastic demand. The optimality of a signal that generates unit-elastic demand is also true in Choi et al. (2019), who look at consumer-optimal signals in a monopoly problem for search goods (where, in contrast to this paper, true values are apparent upon inspection instead of through consumption). These results are closely related to those of Roesler and Szentes (2017), Condorelli and Szentes (2020), and Yang (2019), who all establish that truncated Pareto distributions over valuations–which correspond to unit-elastic demand–are optimal for consumers in variants of a bilateral trade setting.

There are also several papers that explore competition through information provision when there are no search frictions ($c = 0$). The unique equilibrium in Spiegler (2006) is essentially isomorphic to the specific case in the frictionless model when the mean is fixed at 1/2. Boleslavsky and Cotton (2015) and Albrecht (2017) both derive results that characterize the the two-player solution to this problem of frictionless competitive information provision. Whitmeyer (2018) looks at a dynamic version of the two-player game, and Au and Kawai (2019) modify the two-player scenario to allow for arbitrary correlation between the senders' qualities. Special mention is due to Au and Kawai (2020), who establish the unique (symmetric) equilibrium for the n-player game. These results were derived independently in an earlier version of this paper, using different techniques. Koessler et al. (2017) look at a general setting in which multiple persuaders provide information about their own dimension of some multidimensional state.

A natural point of comparison for this paper is the collection of papers that explore price-directed search. One important early foray in the area is Armstrong and Zhou (2011), who allow firms to post prices in a modified Hotelling environment with search frictions. A subsequent major contribution is Choi et al. (2018), who incorporate (posted) price competition into a model of Weitzman search. With advertised prices, both papers find that as search costs increase, prices decrease and consumer surplus may thus increase. This relationship is also uncovered by Ding and Zhang (2018), who add product differentiation and posted prices to the setting of Stahl (1989); and by Haan et al. (2018), who allow for both posted prices and (publicly) observable product characteristics. Armstrong (2017) also discusses the same pattern in his overview on price competition in an ordered consumer search setting.

In each such paper on price-directed consumer search, there is a tension inherent to firms' pricing decisions. Setting a lower price makes a firm more likely to be visited early as well as make a sale if visited, yet obviously lowers the firm's profit directly. As search frictions increase, it becomes more important to attract (and retain) consumers, which drives prices down. At first glance, this trade-off seems like a direct analog of the persuasion/attraction conflict in this paper. However, there are important differences. In particular, note that lowered prices help with both persuasion and attraction–consumers are both more likely to visit *and* to stop as a firm lowers its price. The idea that firms can increase the chance of being selected if visited, at the expense of being visited in the first place, is completely absent from this literature that focuses on pricing; yet is the driving feature in our model. To put differently, in both price and information-directed competition in consumer search, the incentive to be visited early is fundamental, yet the costs of such prominence are different.

The vein of research that focuses on *obfuscation* is also relevant. Two such papers are Ellison and Wolitzky (2012) and Ellison and Ellison (2009). In the first, the authors extend the model of Stahl (1989) by allowing firms to choose the length of time it takes for consumers to learn its price. Allowing for such delays hurts consumers, since obfuscation leads to longer search times and higher prices. Ellison and Ellison (2009), in turn, provide empirical evidence suggesting that as technology has made price search easier for consumers, firms have responded by taking actions that make price search more difficult. More recently, Gamp and Krähmer (2017) examine a scenario in which sellers can dupe naive consumers into buying products that are lemons. As in this model (though through a different mechanism), search frictions can be beneficial to consumers.

Finally, just as search frictions can improve consumer welfare (when prices are posted), so too can other types of frictions, which has been observed by some papers from the rational inattention literature. The forces driving these results are quite different than the attraction/persuasion trade-off that we explore here. De Clippel et al. (2014) look at a model in which consumers have demand for multiple goods, but can only compare prices for a fraction of them. Consumers have a default seller (the market leader) for each good and for each can choose whether to explore the market further and compare prices to a competitor or simply purchase from the leader. Because consumers can only explore the markets for a subset of goods, there is an incentive for leaders to lower prices and therefore stay "under consumers' radar." In contrast, firms in our setting compete to get "on consumers' radar." In a bargaining scenario in which the buyer is rationally inattentive, Ravid (2020) finds that attention costs strictly benefit a buyer, who obtains bargaining power as a result of his inattention. Conversely, our finding that the consumer may benefit from a positive search cost hinges crucially on market competition. If there is a single firm in our setting, search frictions do not benefit the consumer.

2 The Model

There are $n + 1$ players: one consumer, C, and n *ex-ante* identical single-product firms indexed by i. Each firm's product has a random quality (or match value) to the consumer of either 0 or 1. These qualities are identically and independently distributed, with $\mu \in (0, 1)$ being the prior probability that the quality realization is 1. The consumer needs at most one unit of the good, and her *ex-post* payoff of consumption is normalized to the quality of the product consumed. For simplicity, we assume that the consumer has an outside option with a value no larger than 0.[5] A firm receives a payoff normalized to 1 if the consumer picks its product, and 0 otherwise. All players are expected-payoff maximizers.

At the beginning of the game, neither the consumer nor the firms know the quality realizations of their products. Each firm simultaneously commits to a signal, which is a measurable function $\pi_i : \{0, 1\} \longrightarrow \Delta(S)$ with some space of signal realizations S. The primary focus of this paper is the scenario in which the chosen signals are publicly posted, and therefore shape the consumer's behavior directly.[6]

[5] In Section 6.3, we show that our findings continue to hold when this assumption is relaxed.

[6] In Section 5.2 we study a setting in which the chosen signals are hidden, and can only be observed after incurring the search cost. Naturally, in this case, the consumer conjectures firms' signal choices, which must be correct at equilibrium.

Guided by the informativeness of the posted signals, the consumer learns about the firms' product qualities by visiting the firms and observing their signal realizations in sequence. Each such inspection requires a search cost of $c > 0$. After observing the signal realization of a firm, the consumer updates her prior to the posterior expected quality of the product offered by that firm. At any stage of her sequential search, the consumer can stop her search by buying from any previously visited firm; recall is assumed to be free. Alternatively, she can continue her search by visiting more firms or stop her search by collecting the outside option. To avoid trivial cases, we assume throughout that $c \leq \mu$, so that search is not strictly dominated for the consumer.

The solution concept used is subgame-perfect Nash equilibrium. Given the *ex-ante* symmetry of the firms, it is natural to focus on symmetric equilibria in which (i) all firms adopt a common (possibly mixed) strategy, and (ii) the consumer adopts a tie-breaking rule that treats all firms identically.

3 Preliminary Analysis

The purpose of this section is to simplify the game set up in the last section, and we proceed through a number of steps. First, we note that a firm's strategy space can be expressed as the set of distributions over posterior qualities and that the consumer's sequentially optimal search strategy takes a simple form. Second, we explain the necessity of considering mixed strategies by noting that there are no pure-strategy equilibria in a large region of the parameter universe. Finally, using the fact that the consumer's decision can be expressed as a static discrete choice problem, we show how the strategic interaction between the firms can be condensed and simplified.

3.1 Basics

This subsection reformulates the game set up in Section 2 into one of competition in the design of distributions over posteriors. Instead of choosing signals directly, the strategy space of each firm can be redefined without loss to be the set of feasible distributions over posterior (expected) qualities. To that end, we introduce the following definition.

Definition 3.1. A distribution over posteriors, F, is **Feasible** if it has mean μ and its support is a subset of $[0,1]$. $\mathcal{M}_{[0,1]}(\mu)$ denotes the set of feasible distributions.[7]

As a signal realization affects payoffs only through its implied (posterior) expected product quality, it is a standard result in the literature that we may define a firm's pure strategy to be a feasible distribution over posterior qualities. Consequently, a generic mixed strategy of a firm is a randomization over the set of feasible distributions, and the set of mixed strategies is $\Delta\left(\mathcal{M}_{[0,1]}(\mu)\right)$. Note that if a firm plays a mixed strategy, its pure strategy realization occurs and becomes public *before* the consumer begins her search.

Next, given the n posted distributions over posterior qualities, the sequentially optimal selection and stopping rule has been identified by Weitzman (1979). A brief recap of his finding is useful. For any distribution F_i chosen by Firm i, define the corresponding reservation value, $U(F_i)$, implicitly as the solution to the following equation (in u):

$$c = \int_u^1 (x-u)\,dF_i(x)\,. \tag{1}$$

The set of feasible reservation values $\{U(F)\}_{F\in\mathcal{M}_{[0,1]}(\mu)}$ is bounded between $\underline{U} \equiv \mu - c$ and $\bar{U} \equiv 1 - c/\mu$. The lower bound is induced by any feasible distribution whose support is entirely (weakly) above $\mu - c$, one of which is the degenerate distribution at μ (which corresponds to a completely uninformative signal). The upper bound is uniquely induced by the feasible distribution supported on $\{0,1\}$ (which corresponds to a fully revealing signal). It is not difficult to see that any intermediate reservation value can be achieved by some feasible distribution, so $\{U(F)\}_{F\in\mathcal{M}_{[0,1]}(\mu)} = \left[\underline{U}, \bar{U}\right]$.[8] Moreover, the reservation value rewards informativeness: for any pair of feasible distributions F and G, if F is a mean-preserving contraction of G, then $U(G) \geq U(F)$.

The optimal strategy of the consumer (a.k.a. Pandora's rule) is as follows.

- **Selection rule:** If a firm is to be visited and examined, it should be the unvisited firm with the highest reservation value.
- **Stopping rule:** Search should be stopped whenever the maximum reservation value of the unvisited firms is lower than the maximum sampled reward or the outside option.

[7]In general, beyond the binary prior case that we explore in this paper, the set of feasible distributions is the set of all mean-preserving contractions of the prior distribution.

[8]For instance, by some convex combination of the two distributions above.

As we focus on symmetric equilibria, we select the (sequentially) optimal strategy in which the consumer adopts a fair tie-breaking rule throughout the search process. With the consumer's search behavior pinned down, we can restrict our attention to the strategic interaction between the firms in our subsequent analysis.

3.2 The Full Information Equilibrium

This subsection focuses on pure-strategy equilibria and identifies the conditions under which one exists as well as the form it takes. We begin with a simple observation:

Lemma 3.2. *There exist no symmetric pure strategy equilibria in which any reservation value $U < \bar{U}$ is induced.*

The intuition mirrors that of the Bertrand model of homogeneous goods in which pricing above marginal cost cannot be sustained in equilibrium. Just as a firm can "undercut" its rivals' marked-up price to obtain a discrete jump up in its demand; here, a firm can provide slightly more information than its rivals' partial revelation.[9] Doing so grants the firm a considerable edge on its competition (since it will move to the top of the consumer's search order), at a negligible loss of persuasion effectiveness. This results in a discrete gain in expected profit, so a symmetric partial-information equilibrium cannot be sustained. This leaves full information as the only candidate pure-strategy equilibrium.

Proposition 3.3. *Define $\bar{\mu} \equiv 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$. A symmetric equilibrium in pure strategies exists if and only if $\mu \geq \bar{\mu}$; i.e., the average quality, μ, is sufficiently high or the number of firms, n, is sufficiently large. In this equilibrium, all firms provide full information.*

This proposition details precisely the conditions under which the attraction incentive dominates the persuasion incentive for the firms. A high μ implies persuasion is likely to succeed, making it paramount for a firm to entice the consumer into visiting it–a failure to do so means the consumer is likely to stop her search at one of a firm's rivals before ever reaching it. A similar

[9] More specifically, suppose all firms but Firm i adopt a common strategy, F, with $U(F) < \bar{U}$. A profitable deviation for Firm i is to modify F by increasing the probability of values 0 and 1 by an arbitrarily small amount and decreasing the probability of a commensurate measure of interior values.

effect is at work if the number of rival firms is large–a low rank in the consumer's search order means a firm is unlikely to ever be visited, let alone make a sale. Consequently, in these cases, the attraction incentive dominates. If rival firms are expected to reveal full information, any departure from it guarantees the deviator the bottom rank in the consumer's search order. With a high average quality and/or large number of competitors, the likelihood that the consumer ever visits that firm is too low for the deviation to be profitable, and the full-information equilibrium can be supported.

The relative importance of attraction in relation to persuasion is smaller if μ and/or n is relatively low. Even if all of its rivals provide full information, an individual firm may find it profitable to provide less information: despite being ranked last, the firm still has a decent chance of eventually being visited. In this case, the attraction motive no longer dominates, and both the attraction and persuasion roles of signals play a part in shaping the equilibrium outcome. By Lemma 3.2, the equilibrium, if it exists, necessarily involves mixed strategies, making its characterization more involved. In the next subsection, we make use of the remarkable discovery of Choi et al. (2018) and Armstrong (2017), who show that the consumer's sequential search can be formulated as a (static) discrete choice problem, which enables us to characterize and establish the uniqueness of the symmetric equilibrium in mixed strategies in a tractable way.[10]

3.3 Reformulating the Game and Main Analysis

Using the observation that the consumer's optimal shopping strategy affects firms' payoffs only through her eventual purchase decision (rather than the details of the exact search paths), the strategic interaction between firms can be modelled as the competition over the realizations of effective values. The **Effective Value** of Firm i is defined as $W_i \equiv \min\{p_i, U(F_i)\}$, where p_i is the realized posterior quality, and F_i is the distribution over posteriors chosen by Firm i. After distribution F_i is chosen, but before the posterior realizes, Firm i's effective value, W_i, is a random

[10] As it turns out, the full disclosure equilibrium exposed in this subsection corresponds to a special case of a more general result that we derive in the next section. The main result there, Proposition 4.5, establishes that the (essentially) unique equilibrium of the game (for all parameter values) must take a particular linear form, of which the full disclosure equilibrium serves as avatar when μ is sufficiently high.

variable with distribution

$$H(w;F_i) \equiv \begin{cases} F_i(w) & \text{if } w < U(F_i) \\ 1 & \text{if } w \geq U(F_i) \end{cases}. \tag{2}$$

As shown in Choi et al. (2018), the consumer eventually purchases from the firm with the highest effective value realization, provided that it is no less than her outside option. Therefore, a firm's problem can be stated as choosing an effective-value distribution that maximizes the probability of realizing the highest effective value among all of the competing firms (including the consumer's outside option).

This begs the question, what distributions over effective values can a firm induce? Each mixed strategy $\sigma \in \Delta\left(\mathcal{M}_{[0,1]}(\mu)\right)$ of a firm induces a distribution G_σ over reservation values by $G_\sigma(w) \equiv \sigma(\{F : U(F) \leq w\})$. For any reservation value U' in the support of G_σ, there is a profile of feasible distributions over posteriors that attain this reservation value $\{F \in supp(\sigma) : U(F) = U'\}$ in the support of σ. Denote by $F_{\sigma,U}$ the implied distribution over posteriors conditional on the realization of reservation value U under the mixed strategy σ. It is without loss to replace the profile of feasible distributions above with $F_{\sigma,U}$ in the mixed strategy σ, as doing so leaves the induced distribution over effective values unaffected. Consequently, a generic mixed strategy takes the form $\left(G(\cdot), \{F_U(\cdot)\}_{U \in supp(G)}\right)$, for some reservation-value distribution G and a feasible distribution over posteriors $F_U(\cdot)$ that attains each $U \in supp(G)$. With this convention of representing a mixed strategy, the effective-value distribution induced by Firm i's mixed strategy is given by

$$\begin{aligned} H_i(w) &\equiv \Pr(\min\{p_i, U(F_i)\} \leq w) \\ &= \Pr(U(F_i) \leq w) + \Pr(p_i \leq w < U(F_i)) \\ &= G(w) + \int_w^{\bar{U}} F_s(w)\, dG(s). \end{aligned} \tag{3}$$

Armed with this, we introduce another definition.

Definition 3.4. A distribution over posterior effective values, H_i, is **Inducible** if there exists a mixed strategy $\left(G(\cdot), \{F_U(\cdot)\}_{U \in supp(G)}\right)$ such that Equation (3) holds. $\mathcal{I}_{[0,1-c/\mu]}(\mu - c)$ denotes the set of inducible distributions of effective values.

The following observation may be obvious, yet deserves to be stated for clarity.

Remark 3.5. Let $\mathscr{P}_{[0,1-c/\mu]}(\mu - c)$ denote the set of distributions over effective values that can be induced by a deterministic choice of feasible distribution F; that is, that can be induced by a firm's pure strategy. Then there are distributions over effective values supported on $[0, 1 - c/\mu]$ with mean $\mu - c$ that cannot be induced; and there are inducible distributions over effective values that cannot be induced by pure strategies. In notation,

$$\mathscr{P}_{[0,1-c/\mu]}(\mu - c) \subset \mathscr{I}_{[0,1-c/\mu]}(\mu - c) \subset \mathcal{M}_{[0,1-c/\mu]}(\mu - c)\,.$$

We should take two things from this remark. First, although it is tempting, we may not treat a firm's problem as a standard persuasion problem in which it just chooses a Bayes-plausible distribution over effective values, as the optimal distribution from the set $\mathcal{M}_{[0,1-c/\mu]}(\mu - c)$ may not be inducible. Indeed, as we will see in the next subsection, this inducibility restriction does have bite, at least in some parameter configurations. Second, the remark illustrates that equilibria may require firms to mix over feasible distributions of posteriors. Again, we will discover in the next section that for some parameter values, this is necessary in a symmetric equilibrium.

Proof. The weak inclusion of these sets is trivial and a pair of examples suffices to show that the inclusion is strict. As noted above, the maximal reservation value $1 - c/\mu$ is uniquely induced by the Bernoulli distribution. Let S_p denote the following (parameterized) binary distribution over effective values:

$$S_p = \begin{Bmatrix} \frac{(\mu-c)(\mu-p)}{\mu(1-p)} & 1 - \frac{c}{\mu} \\ 1 - p & p \end{Bmatrix},$$

where the top row is the support of the distribution and the bottom row the associated probability weights (the pmf). Evidently, $S_p \in \mathcal{M}_{[0,1-c/\mu]}(\mu - c)$ for any $p \in [0, \mu]$; whereas $S_p \notin \mathscr{I}_{[0,1-c/\mu]}(\mu - c)$ for all $p \neq \mu$.

Next, let P and Q be the following two binary distributions over effective values (with $\mu = 1/2$ and $c = 1/8$):

$$P = \begin{Bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{Bmatrix}, \quad \text{and} \quad Q = \begin{Bmatrix} \frac{1}{4} & \frac{15}{24} \\ \frac{2}{3} & \frac{1}{3} \end{Bmatrix}.$$

Clearly, both $P, Q \in \mathscr{P}_{[0,1-c/\mu]}(\mu - c)$. The following ternary distribution over effective values, R, can be induced by randomizing fairly between P and Q.

$$R = \begin{Bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{15}{24} \\ \frac{7}{12} & \frac{1}{4} & \frac{1}{6} \end{Bmatrix}.$$

Thus, by construction $R \in \mathcal{I}_{[0,1-c/\mu]}(\mu - c)$. However, the unique pure strategy distribution over values that induces R is

$$R' = \begin{Bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{11}{8} \\ \frac{7}{12} & \frac{1}{4} & \frac{1}{6} \end{Bmatrix},$$

which is infeasible. Accordingly, $R \notin \mathcal{P}_{[0,1-c/\mu]}(\mu - c)$. ∎

The following property of inducible effective-value distributions follows from the definitions of the reservation-value equation (1) and the effective-value distribution (2), and is useful in our subsequent analysis.

Lemma 3.6. *Let F be a distribution over posteriors with mean μ and reservation value U. Its induced effective-value distribution has mean $\underline{U} = \mu - c$. Moreover, the expected effective value conditional on falling short of U lies in the interval $[0, \bar{a}(U)]$, where $\bar{a}(U) \equiv \frac{\mu - c - \mu U}{1 - c - U}$.*

The preliminary analysis above explains how our game can be cast as competition over inducible effective-value distributions. The optimization program over inducible effective-value distributions will therefore play a crucial role in the equilibrium analysis. The next subsection proposes a graphical solution to this problem.

3.3.1 Finding the Optimal Inducible Effective-Value Distribution by Concavification

In this subsection, we show how the concavification approach by Kamenica and Gentzkow (2011) can be adapted to a firm's problem of finding the optimal inducible[11] effective-value distribution. We focus temporarily in this subsection on the optimization problem of a single firm whose payoff as a function of its realized effective value is $\Pi : [0, \bar{U}] \longrightarrow \mathbb{R}$.

The optimal effective-value distribution can be found in two steps. We first identify the optimal effective-value distribution for each implied reservation value $U \in [\underline{U}, \bar{U}]$. The overall optimum can then be found by comparing the expected payoffs for each reservation value.

For each reservation value $U \in [\underline{U}, \bar{U}]$, denote by $\Pi_U : [0, U] \longrightarrow \mathbb{R}$ the restriction of Π to the domain $[0, U]$, and denote by $\hat{\Pi}_U : [0, U] \longrightarrow \mathbb{R}$ the concave closure of Π_U. The optimal effective-value distribution conditional on reservation value U can be found by solving the

[11] Henceforth, to save space we omit this modifier.

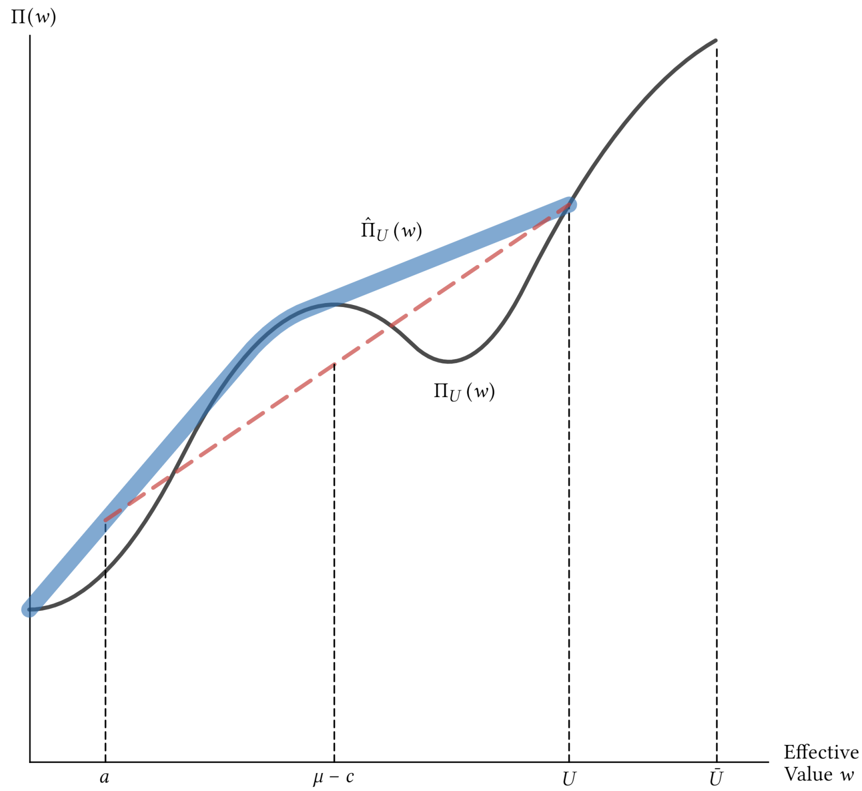


Figure 1: An illustration of Π; its restriction to $[0, U]$, Π_U; and the concave closure of Π_U, $\hat{\Pi}_U$.

following problem:

$$V(U) \equiv \max_{a \in [0, \bar{a}(U)]} \Pi(U) - \frac{\Pi(U) - \hat{\Pi}_U(a)}{U - a}[U - \underline{U}], \tag{4}$$

where $\bar{a}(\cdot)$ is defined in Lemma 3.6. The intuition is as follows. Observe that a distribution over posteriors may be decomposed into the following three components: (i) a conditional distribution over $[U, 1]$, (ii) a conditional distribution over $[0, U]$, and (iii) the relative weight over these two regions of posteriors. As component (i) has no impact on the effective-value distribution and hence the firm's payoff, it suffices to focus on the choices of the latter two components.

The optimal choice of component (ii) can be characterized by the concavification approach of Kamenica and Gentzkow (2011): the optimal conditional distribution over $[0, U]$ can be found by identifying the concave closure $\hat{\Pi}_U$ of the payoff function restricted to the domain $[0, U]$. Evidently, if it has a conditional mean $a \in [0, \bar{a}(U)]$, then the maximized conditional payoff is $\hat{\Pi}_U(a)$. Component (iii), the relative weight over the upper and the lower posterior regions, is uniquely pinned down by the choice of $a \in [0, \bar{a}(U)]$ and the reservation-value equation (1). Specifically, if F is a feasible distribution over posteriors with reservation value U and a mean a

conditional on the posterior falling short of U, it is necessary that $F(U) = (U - \underline{U}) / (U - a)$.[12]

Using the observations above, the optimal effective-value distribution can be found by choosing the conditional mean $a \in [0, \bar{a}(U)]$ to maximize

$$\hat{\Pi}_U(a) \times F(U) + \Pi(U) \times (1 - F(U)) = \hat{\Pi}_U(a) \times \frac{U - \underline{U}}{U - a} + \Pi(U) \times \frac{\underline{U} - a}{U - a}. \tag{5}$$

This problem is equivalent to that of (4). A graphical illustration is shown in Figure 1.

Having identified the optimal effective-value distribution for each reservation value, the overall optimum can be found by optimizing $V(U)$ over $U \in \left[\underline{U}, \bar{U}\right]$.[13] The following proposition summarizes the graphical approach for effective-value distribution optimization discussed above.

Proposition 3.7. *The firm's maximized payoff is given by* $\max_{U \in \left[\underline{U}, \bar{U}\right]} V(U)$. *Moreover, suppose* $U^* \in \left[\underline{U}, \bar{U}\right]$ *maximizes* $V(\cdot)$ *and* $a^* \in [0, \bar{a}(U^*)]$ *maximizes* $\Pi(U^*) - \frac{\Pi(U^*) - \hat{\Pi}_{U^*}(a)}{U^* - a}(U^* - \underline{U})$. *An optimal effective-value distribution has a mass of* $\frac{\underline{U} - a^*}{U^* - a^*}$ *assigned to* $\frac{(\mu - a^*)U - ca^*}{\underline{U} - a^*}$ *(which is above* U^**) and the residual mass* $\frac{U^* - (\mu - c)}{U^* - a^*}$ *assigned to values below* U^* *according to the construction of the concave closure* $\hat{\Pi}_{U^*}$ *at* a^*.

4 The Symmetric Equilibrium in Competition over Effective Values

In this section, we explicitly characterize the symmetric equilibrium by analyzing the game of effective-value competition set up in the previous section. We show that the symmetric-equilibrium distribution of effective values is necessarily unique, and implies a specific linear

[12]To see this, note that the reservation-value equation (1) implies that the expected value conditional on exceeding U is $\int_U^1 x dF(x) / [1 - F(U)] = U + c / [1 - F(U)]$. The conditional means below and above U, together with the feasibility requirement that the unconditional mean equals μ, then fully determine the relative weights over the two posterior regions as follows:

$$a \times F(U) + \left(U + \frac{c}{1 - F(U)}\right) \times (1 - F(U)) = \mu.$$

[13]The formula in (5) illustrates the trade-off of attraction versus persuasion. If the payoff function is non-decreasing, a higher choice of U increases the attractiveness of the firm. Its probability of realization $(1 - F(U))$ is, however, decreasing in U (holding a fixed), indicating that persuasion is less effective with an aggressive choice of U.

structure of the payoff function (in effective values) facing each individual firm. We establish equilibrium existence by explicitly constructing a mixed strategy that randomizes over binary distributions of posteriors.

Formally, the (reformulated) game is as follows. Each of the n firms simultaneously chooses an inducible effective-value distribution $H_i \in \mathcal{I}_{[0,\bar{U}]}(\underline{U})$, with the objective of maximizing the probability that its realized effective value, w_i, is the highest among those of all the firms (with fair tie-breaking).

We begin with the following preliminary observations about the effective-value distribution in any symmetric equilibrium.

Lemma 4.1. *In any symmetric equilibrium, a firm's effective-value distribution has no atoms except possibly at* 0 *and* $\bar{U}$.

The lemma is quite intuitive. If other firms are placing an atom at some $w \in \left(0, \bar{U}\right)$, it is never optimal for a firm to respond by placing an atom there for the following reason. If w is a reservation value, offering a marginally more informative signal discretely improves the power of attraction. If w is a posterior realization, shifting the weight to a marginally better posterior realization discretely improves the power of persuasion.[14]

The lemma implies that in any symmetric equilibrium, the expected payoff facing an individual firm as a function of its realized effective value, w, takes the following form:

$$\Pi(w; H) \equiv \begin{cases} \frac{H(0)^{n-1}}{n} & \text{if } w = 0 \\ H(w)^{n-1} & \text{if } w \in [0, \bar{U}) \\ \lim_{w' \to \bar{U}^-} \frac{1-H\left(w'\right)^n}{n(1-H(w'))} & \text{if } w = \bar{U} \end{cases}, \tag{6}$$

for some distribution function $H \in \mathcal{I}_{[0,\bar{U}]}(\underline{U})$ that is continuous over $(0, \bar{U})$.

As the consumer eventually purchases from one of the firms (recall that her outside option is assumed to be irrelevant), the firms' competition is a zero-sum game and the *ex-ante* expected payoff of each firm is $1/n$ in any symmetric equilibrium. The crucial step in our equilibrium

[14] This argument does not apply to the effective value 0 because assigning positive weight there corresponds uniquely to full disclosure, a possibility that can arise in equilibrium as we have shown in Section 3.2 (and will rederive below).

characterization is to show that the payoff function $\Pi(w;H)$ facing each firm must possess the linear structure depicted in Figure 2.

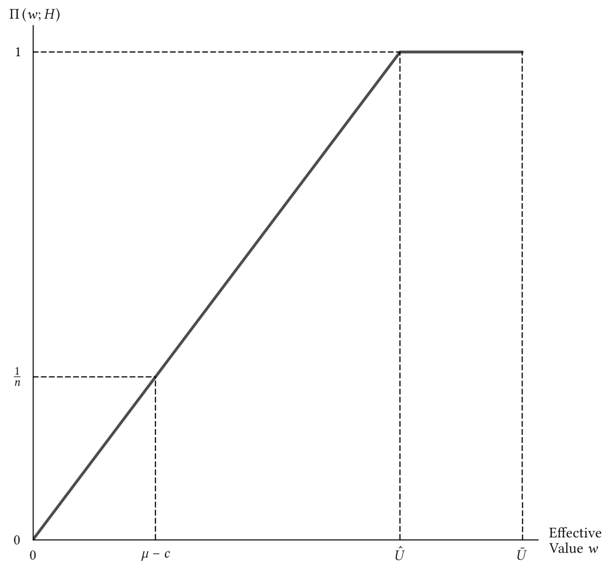
Besides highlighting a key property of the competition over effective values, the linearity of the payoff function significantly helps simplify our search for an equilibrium, as each linear structure is characterized by a pair of scalars that can readily be identified by simple equilibrium conditions.

The sufficiency of the linear structure for a symmetric equilibrium is straightforward. When an individual firm faces a payoff function that has the linear structure above, it is indifferent between offering any reservation value on the support and is thus willing to randomize over the relevant range. Loosely speaking, the linear structure is also necessary because this is the only way to ensure the incentive-compatibility for randomization over an interval of interior effective values. If, over an interval, say $\left[\underline{I},\bar{I}\right]\subset\left(0,\bar{U}\right)$, the payoff function $\Pi(w;H)$ facing an individual firm is convex and non-linear, no positive mass would be assigned to its interior, implying an atom at the boundary points, contradicting Lemma 4.1. If the payoff function is, on the other hand, concave and non-linear over $\left(0,\bar{U}\right)$, firms are willing to put a positive measure close to 0 only if the payoffs of offering reservation values arbitrarily close to $\bar{U}$ all coincide with $1/n$. A straightforward computation shows that this is generically impossible. The following lemma details the exact requirement of the linear structure in the equilibrium payoff function, as well as its necessity for a symmetric equilibrium.

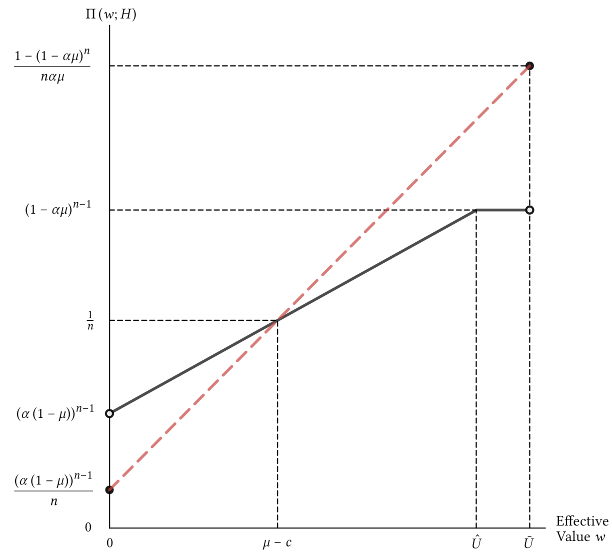
Lemma 4.2. *Suppose the consumer's outside option is irrelevant. Denote by H a symmetric-equilibrium distribution of effective values chosen by each firm, let $\alpha\in[0,1]$ be the probability that a firm offers full information, and let $\hat{U}\equiv\sup\left(supp(H)/\left\{\bar{U}\right\}\right)$. Then the payoff function in effective values facing each individual firm must have the following linear structure:*

$$\Pi(w;H)=\begin{cases}\frac{1}{n}\left(\alpha(1-\mu)\right)^{n-1} & \text{if } w=0\\ \left(\alpha(1-\mu)\right)^{n-1}+\frac{(1-\alpha\mu)^{n-1}-\left(\alpha(1-\mu)\right)^{n-1}}{\hat{U}}w & \text{if } w\in\left(0,\hat{U}\right]\\ (1-\alpha\mu)^{n-1} & \text{if } w\in(\hat{U},\bar{U})\\ \frac{1-(1-\alpha\mu)^{n}}{n\alpha\mu} & \text{if } w=\bar{U}\end{cases},\tag{7}$$

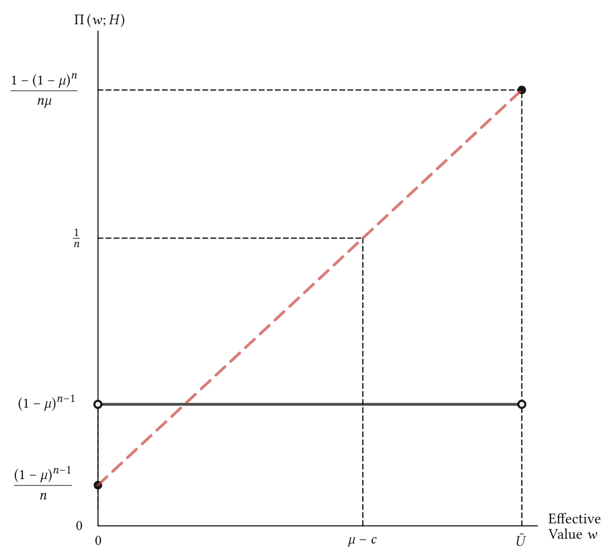
The symmetric-equilibrium distribution of effective values can thus be fully characterized by the atom α at the top and the upper bound $\hat{U}$ of the interior support. Note that full disclosure, i.e.,



(a) The equilibrium payoff when $\mu \leq \underline{\mu}$.



(b) The equilibrium payoff when $\mu \in \left(\underline{\mu}, \bar{\mu}\right)$.



(c) The equilibrium payoff when $\mu \geq \bar{\mu}$.

Figure 2: The linear structure of a firm's payoff function, $\Pi(w; H)$.

a binary effective-value distribution with support $\left\{0, \bar{U}\right\}$, is a special case of the linear structure above with $\alpha = 1$ and $\hat{U} = 0$.

To pin down the equilibrium distribution of effective values, it remains to solve for the parameters α and $\hat{U}$ in (7). Consider first the case of an interior atom at the top, $\alpha \in (0, 1)$. Here, reservation values $\bar{U}$ and $\hat{U}$ are both on the support, and hence must deliver the equilibrium payoff of $1/n$. Moreover, the linearity of the payoff function implies that the optimal payoff of adopting reservation value $\hat{U}$ is $\Pi\left(\underline{U}; H\right)$. Consequently,

$$\frac{1}{n} = \Pi\left(\underline{U}; H\right), \text{ and} \tag{8}$$

$$\frac{1}{n} = \frac{\underline{U}}{\bar{U}} \times \Pi\left(\bar{U}; H\right) + \left(1 - \frac{\underline{U}}{\bar{U}}\right) \times \Pi\left(0; H\right). \tag{9}$$

If, on the contrary, α takes on the extreme values of 0 and 1 in equilibrium, only one of the equations above holds. In the case of no atom at the top, i.e., $\alpha = 0$, $\hat{U}$ is on the support while $\bar{U}$ is not. Therefore, equation (8) is still necessary, but the right-hand side of (9) can fall below $1/n$. In the case of full disclosure in equilibrium, i.e., $\alpha = 1$, equation (9) is necessary, but the right-hand side of (8) can fall below $1/n$.

After substituting (7) into (8) and (9), the observations above reduce the quest for a symmetric equilibrium into solving a system of at most two equations in two unknowns. The following lemma explicitly states the unique solution of the system.

Lemma 4.3. *Except for the knife-edge case in which $\mu = 1/2$ and $n = 2$, the system (8) and (9) has a unique solution in $\left(\alpha, \hat{U}\right)$ that depends on the average product quality μ and the total number of firms n as follows. Recall $\bar{\mu} \equiv 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$ and define $\underline{\mu} \equiv \frac{1}{n}$.*

(a) If $\mu \leq \underline{\mu}$, then $\alpha = 0$ and $\hat{U} = n\underline{U}$.

(b) If $\mu \in \left(\underline{\mu}, \bar{\mu}\right)$, then $\alpha \in (0, 1)$ is the unique solution to

$$(1 - \alpha\mu)^n - (\alpha(1 - \mu))^n = 1 - \alpha, \tag{10}$$

and

$$\hat{U} = \frac{(1 - \alpha\mu)^{n-1} - (\alpha(1 - \mu))^{n-1}}{n^{-1} - (\alpha(1 - \mu))^{n-1}} \underline{U}. \tag{11}$$

(c) If $\mu \geq \bar{\mu}$, then $\alpha = 1$ and $\hat{U} = 0$.

If $\mu = 1/2$ and $n = 2$, then cases (a)-(c) coincide, and there exists a continuum of solutions in which α takes any value in $[0, 1]$ and $\hat{U}$ is given in (11).

Making use of the necessary conditions for a symmetric equilibrium, Lemmata 4.2 and 4.3 together pin down the unique payoff function that can arise in equilibrium. The distribution of effective values that generates this payoff function can be readily recovered using (6):

$$H(w)=\begin{cases}\alpha(1-\mu) & \text{if } w=0\\ \left((\alpha(1-\mu))^{n-1}+\frac{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}{\hat{U}}w\right)^{\frac{1}{n-1}} & \text{if } w\in\left(0,\hat{U}\right]\\ 1-\alpha\mu & \text{if } w\in(\hat{U},\bar{U})\\ 1 & \text{if } w=\bar{U}\end{cases}. \tag{12}$$

The linearity of the payoff function (7) implies that the effective-value distribution H above delivers an expected payoff of $1/n$, as required. It remains to show that this distribution is indeed inducible, i.e., that there exists a mixed strategy over distributions over posteriors that implies the effective-value distribution (12). Establishing the inducibility of distribution H ensures that it is indeed a mutual best response, thus giving a symmetric equilibrium. The following lemma explicitly constructs such a mixed strategy, which involves randomization over binary distributions over posteriors only.

Lemma 4.4. *The effective-value distribution H in (12), with α and $\hat{U}$ given by Lemma 4.3, is inducible. Moreover, it can be generated by a mixed strategy $\left(G(\cdot),\{F_U(\cdot)\}_{U\in supp(G)}\right)$ in which F_U is binary for each $U\in supp(G)$.*

While Lemma 4.4 identifies a particularly simple mixed strategy that implements the equilibrium effective-value distribution, this is not the unique implementation. Therefore, while we establish uniqueness of the effective-value distribution in equilibrium, the mixed strategy that generates it may not be unique.

The binary implementation described in this lemma has several interesting properties. Once the initial randomness from the firms mixing is resolved, the consumer is faced with n firms, each with unique binary distributions that nest within each other, like a matryoshka doll. Evidently, the experiments chosen by the firms can be ranked according to the Blackwell order, and, on path, the consumer searches them in order of their Blackwell informativeness. The consumer stops only if she observes the high realization at a firm. Otherwise, she continues her search, and selects the last firm no matter its realization. Though this is a search in which recall is allowed, the consumer never utilizes this, and *never* returns to a firm from which she had previously moved on.

We see that a scenario arises endogenously that allows for a much simpler (optimal) search protocol than Pandora's Rule. Namely, the consumer merely visits the firms in order of informativeness, and selects a firm if its high value is realized–indeed it is obvious that she should stop since she knows that she will not see a higher realization at any of the remaining firms. If she reaches the last firm, she selects that firm with certainty.

The analysis above is summarized by the following proposition.

Proposition 4.5. *Except for the knife-edge case in which $\mu = 1/2$ and $n = 2$, there exists a unique symmetric equilibrium in the firms' competition over effective values. If $\mu = 1/2$ and $n = 2$, there exists a continuum of symmetric equilibria. The equilibrium effective-value distribution gives rise to a payoff function with a linear structure as in (7).*

The linearity of the payoff function (7) highlights how the trade-off between attraction and persuasion in firm's signal design problem is balanced in equilibrium. Choosing a more informative signal, and hence a high reservation value, U, facilitates the attraction of the consumer. The result of more aggressive disclosure, corresponding to a higher choice of U, is a higher likelihood that the consumer will pay the firm a visit, which is indicated by the strict increase of the payoff function (7) over $\left[\underaccent{\bar}{U}, \hat{U}\right]$. Increasing one's attractiveness, however, comes at the expense of persuasion effectiveness. Specifically, the probability of realizing U as the effective value, and hence the likelihood of converting a visit into a sale, is decreasing in the choice of U, as the mean of any inducible effective-value distribution is fixed at $\underaccent{\bar}{U}$ (recall Lemma 3.6). The linearity of the payoff function (7) above and below $\underaccent{\bar}{U}$ implies that the aforementioned benefit and cost cancel out exactly in equilibrium and that the firm is indifferent between offering a range $\left[\underaccent{\bar}{U}, \hat{U}\right]$ of reservation values.

Figure 2 depicts the three possible forms a firm's payoff function may take and thereby indicates the three possible guises of the equilibrium distributions. If the average quality is sufficiently low, $\mu \leq \underaccent{\bar}{\mu}$, firms choose atomless distributions over effective values such that the distribution of the maximum realized effective value that any one firm faces is the uniform distribution on $n\underaccent{\bar}{U}$. On the other hand, if the average quality is sufficiently high, $\mu \geq \bar{\mu}$, firms provide full information.

Finally, if $\mu \in \left(\underline{\mu}, \bar{\mu}\right)$, the equilibrium distributions still yield a linear payoff on an interior interval but also have atoms at 0 and $\bar{U}$.

We conclude this section by discussing the effect of the search cost, c, on the consumer's welfare. While an increase in c has a direct negative effect on the consumer's payoff, it induces more intense competition between the firms, which could potentially benefit the consumer. In fact, in the price-competition setting of Choi et al. (2018), the indirect effect of intensified competition can be so strong that the consumer surplus increases with a higher search cost. In contrast, we find that when the search cost, c, is positive, the indirect effect can only partially offset the direct negative effect, and the consumer still suffers from an increase in c.

Corollary 4.6. *For $c > 0$, an increase in c worsens the equilibrium distributions over effective values in the sense of first-order stochastic dominance, thus hurting the consumer's ex-ante welfare.*

5 Two Benchmarks

In our main model, the consumer can learn the firms' posted signals for free, but discovering their realizations is costly. Consequently, a firm's posted signal plays the dual role of attracting the consumer to visit it and persuading her to purchase from it. The role of attraction is key in driving both our full disclosure and information dispersion results. Namely, it ensures that partial disclosure cannot be supported in a symmetric equilibrium because marginally overbidding one's rivals' reservation value results in a certain visit by the consumer and hence a discrete increase in the probability of a sale. Moreover, the attraction incentive intensifies the competition between the firms, resulting in more informative disclosure. To illustrate the role of attraction in driving the results above, we consider two alternative scenarios. In the first, the consumer can learn both the posted signals and their realizations for free. In the second, the signals are no longer posted, so the consumer only discovers a firm's signal and its realization after paying the search cost. In contrast to our main model, neither setting generates information dispersion as the unique equilibrium outcome. Moreover, we show that equilibrium disclosure lessens, possibly radically, when the signals do not play a role in enticing the consumer to visit.

5.1 Costless Signal Realizations

This subsection considers the case in which $c = 0$, so that the consumer can observe the firms' signals *and* their realizations at no cost. Crucially, the limit game (with $c = 0$) is qualitatively different from the limiting game (with an arbitrarily small but positive c). Regardless of how small c is, as long as it is positive, the consumer must discover the firms' signal realizations in sequence, and this engenders the attraction motive. In contrast, when $c = 0$, the consumer has simultaneous and free access to all signal realizations, and the attraction force vanishes. This leads to drastically different equilibrium outcomes. First, with $c = 0$, a symmetric pure-strategy equilibrium is possible. Second, the (symmetric) equilibrium distribution of effective values is not always continuous at $c = 0$. When it is not, the informativeness of the equilibrium signal jumps down discretely at $c = 0$. This implies, somewhat counterintuitively, that the consumer may strictly prefer a small positive search cost to no search cost.

When $c = 0$, equation (1) implies that all feasible distributions over posteriors have a reservation value equal to one and hence, each implied effective value coincides with the posterior. As a result, the competition over effective values reduces to a straightforward competition over posterior realizations. This game is studied in Au and Kawai (2020),[15] who show that a unique pure-strategy symmetric equilibrium exists. Intuitively, a pure-strategy equilibrium is possible here because overbidding one's rivals by offering a marginally more informative signal no longer results in an increase in the reservation value. A firm's signal, therefore, has no impact on the likelihood of being inspected by the consumer. As long as the payoff function in posteriors is fully linear, the firm is happy to put positive weight on the range of posteriors over which the payoff function is increasing. As a result, information dispersion is not a necessary feature of the equilibrium.

The difference in nature between the limit game (with $c = 0$) and the limiting game (with an arbitrarily small but positive c) raises the natural question of continuity, which is addressed in the proposition below.

Proposition 5.1. *Let H_c be the equilibrium distribution of effective values when the search cost is $c \in [0, \mu]$, and let H^* be the limiting distribution as the search cost vanishes, i.e., $H_c \longrightarrow H^*$ in*

[15] See also Section 1.1, which references other papers that investigate variants of the frictionless problem.

distribution as $c \longrightarrow 0$.

(i) If $\mu \leq \underline{\mu}$, $H^* = H_0$.

(ii) If $\mu > \underline{\mu}$, H^* *is a mean-preserving spread of* H_0.

It is immediate that the consumer's welfare increases if every firm provides more information. The proposition above thus implies that if $\mu > \underline{\mu}$, the consumer can benefit from a small positive search cost c, which allows her to commit to reward informativeness by visiting firms with a higher reservation value first. This commitment power intensifies the competition in disclosure between the firms, to the benefit of the consumer. In fact, if $\mu \geq \bar{\mu}$, then a positive search cost ensures full disclosure by the firms. In contrast to the classical Diamond paradox (Diamond (1971)); here, a small search cost begets the perfect competition (first-best) level of information provision.

5.2 Hidden Signals

This subsection considers the case in which the firms' signals are not directly observable to the consumer at the outset of her search. In particular, she must incur search cost $c > 0$ to discover both a firm's signal and its realization. Accordingly, a firm's choice of signal cannot affect the consumer's search order. Similar to the benchmark in the previous subsection, this scenario is one of pure persuasion and with the signal's role of attraction evaporated, it is natural to expect that the informativeness of firms' signals should decrease. In fact, we find that the unobservability of signals poses a severe holdup problem akin to the Diamond paradox: each firm has an incentive to secretly lower the information content of its signal to increase the chance of successful persuasion once the consumer has paid it a visit. This begets a stark equilibrium outcome: each firm's signal is uninformative, and the consumer does not find it worthwhile to actively search.

The reasoning is as follows. As no randomness is resolved until the consumer visits, it is without loss to focus on pure strategies of the firms. Consider a purported equilibrium in which some firms are believed to provide useful information to the visiting consumer, i.e., conditional on the visit, the probability that the consumer stops the search and purchases from the visited firm is less than one. Let $\tilde{F}_i$ denote the consumer's conjecture of firm i's distribution over posteriors and let $\tilde{U}_i$ be the corresponding reservation value. Given the consumer's (correct) belief about the equilibrium strategies of the firms, there exists a cutoff posterior realization, denoted by z_i^*, above which the consumer stops the search and buys the product from Firm i conditional on visiting it.[16] It follows from Pandora's rule (Weitzman (1979)) that $z_i^* \leq \tilde{U}_i$. If $z_i^* \leq \mu$, then Firm i can secure the consumer's purchase (conditional on visit) by a distribution over posteriors supported on $[z_i^*, 1]$; that is, Firm i provides no useful information to the consumer. Conversely, if $z_i^* \in (\mu, \tilde{U}_i]$, then the optimal signal of Firm i assigns no weight to posteriors above z_i^*,[17] making reservation value $\tilde{U}_i$ impossible. As a result, the only equilibrium involves all firms providing useless information to the consumer, who stops her search at the first visited firm. The following theorem summarizes the discussion above.

Theorem 5.2. *Suppose the firms' choice of signals are revealed to the consumer only after she pays the visit cost. In all equilibria, each firm offers a distribution of effective values that is degenerate at* $\underline{U}$*,*[18] *and the consumer buys from the first visited firm with probability one.*

The result can be understood as an informational Diamond paradox–in all equilibria, firms provide only the monopoly level of information, and there is no active consumer search.[19] It illustrates that the assumption of the public posting of signals is crucial in generating information dispersion. The driving force behind Theorem 5.2 is the firms' incentives to secretly pool posterior realizations above the consumer's stopping threshold, thus holding up the consumer by concealing all the information that improves the consumer's search outcome.

6 Extensions

Now let us explore a number of extensions of the basic model. In the first, we construct an asymmetric equilibrium that exists only when there are three or more firms. In the second, we allow for heterogeneity between firms and characterize the equilibria when there are two firms

[16] If the purported equilibrium is symmetric, the value of this cutoff is common for all firms and is simply $\tilde{U}_i$. If the purported equilibrium is asymmetric, this cutoff can be computed as the optimal continuation value of search beyond Firm i and is weakly below $\tilde{U}_i$.

[17] This is a simple consequence of the concavification technique of Kamenica and Gentzkow (2011).

[18] Note that the degenerate effective-value distribution can be induced by any feasible distribution over posteriors supported on $[\underline{U}, 1]$, such as the no-disclosure distribution degenerate at μ.

[19] If the consumer has a binding outside option that exceeds $\mu - c$, the market breaks down completely.

with different means. Lastly, we allow for a positive outside option, to which the consumer can always return, and investigate the comparative statics on firm profit.

6.1 Asymmetric Equilibria

While it is natural to focus on symmetric equilibria (as we did in the main portion of the paper) given the *ex-ante* homogeneity of firms, one might wonder whether there are alternative equilibria that demonstrate heterogeneity in firms' strategies. The following proposition points out that while this is impossible in the two-firm case, asymmetric equilibria may arise in some circumstances.

Proposition 6.1. *If $n = 2$, there exist no asymmetric equilibria. Suppose $n \geq 3$ and*

$$\frac{2(1-\mu)}{n+1-2\mu} \geq (1-\mu)^{n-1} \geq \frac{1}{n}.$$

Then there is an equilibrium in which $n-1$ firms choose the binary distribution with support $\{0, 1-c/\mu\}$ and one firm chooses the distribution H, where

$$H(w) = \frac{w}{2(\mu - c)}, \quad \text{if} \quad w \in [0, 2(\mu - c)].$$

In this equilibrium, $n-1$ firms provide full information, and the nth firm chooses a uniform distribution over effective values. In fact, the distribution over effective values chosen by the nth firm is the same as the equilibrium distribution when there are two firms and μ is low. Surprisingly, this equilibrium yields a strictly higher consumer welfare than the coexisting symmetric equilibrium when the search cost is sufficiently small.[20] While the proposition above does not exhaustively identify all asymmetric equilibria, it suggests that an interesting avenue for future research could be comparing the properties of asymmetric equilibria with the symmetric equilibria on which this paper focuses.

This proposition also contrasts nicely with the results of Armstrong et al. (2009), who show that when firms are symmetric, making a firm prominent lowers consumer welfare. Here, we encounter an equilibrium in which $n-1$ firms are endogenously prominent, yet consumer welfare

[20]The reason is as follows. First, the right hand inequality in the proposition precludes the existence of a symmetric equilibrium with full disclosure. Moreover, as the search cost vanishes, the consumer's welfare in the asymmetric equilibrium identified in the proposition converges to the first-best.

rises despite the asymmetric behavior. This is because, in contrast to Armstrong et al. (2009), in which the non-prominent firms raise prices to the detriment of the consumer, it does not matter to the consumer here what the last firm does.

6.2 Two Heterogeneous Firms

Now let us explore the two-firm scenario when the firms have products with different expected qualities. Without loss of generality, let $\mu_1 \geq \mu_2$. There are four different regions of the parameter universe, each of which begets a different variety of equilibrium outcomes. First, if the gap between the means is large enough–specifically, if the maximum reservation value that firm 2 can induce is weakly less than that that firm 1 can induce–then in all equilibria, firm 1 chooses the degenerate distribution over effective values (corresponding to no information) and firm 2 chooses any distribution over effective values. The consumer visits firm 1 first and selects it for sure.

Next, if the gap between means is not as large and firm 2's mean is not too high ($\mu_2 < 1/2$), then there are two regions in which both firms' payoff functions–and hence both firms' distributions over effective values–have the linear structure that we are familiar with. In both of these regions, firm 2 places an atom on effective value 0, and in one of the regions firm 1 places an atom on firm 2's maximum effective value $(1 - c/\mu_2)$.

Finally, if the gap in means is not large but $\mu_2 \geq 1/2$, then firm 1 chooses a binary distribution over effective values supported on 0 and $1-c/\mu_2$, and firm 2 chooses a piece-wise linear distribution over effective values. This is a similar equilibrium, qualitatively, to the asymmetric equilibrium from the previous subsection. The attraction incentive dominates for firm 1 who is always visited first. Firm 2, on the other hand, is content to "pick up the scraps." It is visited second but always selected by the consumer if visited. Moreover, this equilibrium also shares the same property as its analog when firms are homogeneous. The consumer's payoff converges to the first-best (full information) as the search cost vanishes. Thus, our result from the homogeneous firms setting–that search frictions beget the first-best level of information provided the average quality is sufficiently high–carries over to the heterogeneous firms setting. The theorem below provides a synopsis of our findings.

Theorem 6.2. *(i) If $\mu_1 - c \geq 1 - c/\mu_2$, there is a collection of equilibria in which firm 1 chooses the degenerate distribution over effective values with support $\{\mu_1 - c\}$ and firm 2 chooses any distribution over effective values.*

(ii) If $\mu_2 \leq 1/2$ and $1 - c/\mu_2 \geq 2(\mu_1 - c)$, there is an equilibrium in which firm 1 and firm 2 choose linear distributions over effective values. Firm 2 places a mass point on the effective value 0.

(iii) If $\mu_2 \leq 1/2$ and $2(\mu_1 - c) \geq 1 - c/\mu_2 \geq \mu_1 - c$, there is an equilibrium in which firm 1 and firm 2 choose linear distributions over effective values. Firm 2 places a mass point on the effective value 0, whereas firm 1 places a mass point on the effective value $1 - c/\mu_2$.

(iv) If $\mu_2 \geq 1/2$ and $\mu_1 - c < 1 - c/\mu_2$, there is an equilibrium in which firm 1 chooses the binary distribution over effective values with support $\{0, 1 - c/\mu_2\}$ and firm 2 chooses a distribution over effective values that is piece-wise linear with one discontinuity.

6.3 Relevant Outside Option

While our main analysis has abstracted away the consumer's outside option, the tools we developed can be applied to the setting in which the consumer has a relevant outside option. Suppose the consumer has an outside option $u_0 \in \left(0, \bar{U}\right)$ to which she may always return upon quitting her search.[21] A possible interpretation of the outside option is a common product price that is exogenously determined. With a binding outside option, the game between the firms is no longer zero-sum, as the consumer will refrain from making any purchase if the firms' quality realizations turn out to be less than u_0, an event that we find has a strictly positive probability in all symmetric equilibria. Retracing the steps in Section 4, *mutatis mutandis*, we arrive at the following result.

Proposition 6.3. *Suppose the consumer's outside option is relevant, i.e., $u_0 \in \left(0, \bar{U}\right)$, and that $n \geq 3$. There exists a unique symmetric equilibrium in the firms' competition over effective values. There is a cutoff $\mu^{FD} \in (0,1)$ such that the equilibrium has full information disclosure whenever $\mu \geq \mu^{FD}$. When the symmetric equilibrium has partial disclosure, its effective-value distribution implies a payoff function that is linear over its interior support and can only be induced by a mixed strategy.*

The introduction of a relevant outside option, therefore, allows us to investigate factors that can affect industry profit. Interestingly, we find that industry profit can be hurt not only by an

[21]The condition $u_0 < \bar{U}$ ensures that search is not strictly dominated for the consumer and thus remains relevant.

increase in the consumer's search cost c, but also by an improvement in the average product quality μ. These comparative statics illustrate nicely the signal's dual role of attraction and persuasion. On the one hand, an improvement in μ facilitates persuasion, as it lifts the posterior quality realization on average. In fact, it is easy to see that absent any competition, a firm would unambiguously benefit from having a higher μ, as it would allow the firm to increase the probability of realizing a posterior quality above u_0. On the other hand, with a higher μ, the signal's role as an instrument of attraction becomes ever more important, as the chance that the consumer visits low-ranking firms dwindles. A higher average quality thus incites more aggressive information revelation–which harms profits–by lowering the probability that the consumer makes a purchase. We find that the former effect is more important when μ is relatively low, but the latter effect dominates when μ is relatively high.

A standard prediction from the literature on random consumer search (e.g., Wolinsky (1986) and Anderson and Renault (1999)) is that a higher search cost increases profits, as it raises the likelihood that consumers stop and purchase conditional on visiting a firm, thus softening the market competition. In our setting, the firms' signal choices direct the consumer's search, and an increase in the search cost is bad news for the firms, as the consumer is less willing to visit them in the first place. In equilibrium, firms respond by disclosing more aggressively, which results in a higher likelihood that the consumer takes up her outside option, and thus a lower industry profit.[22]

Furthermore, the tension between attraction and persuasion has interesting implications concerning the impact of the average quality, μ, on the firms' profits. On the one hand, a higher μ makes persuasion easier: *ceteris paribus*, the likelihood of a signal realization more favorable than the consumer's outside option goes up with a higher μ, thus raising industry profit. On the other hand, as noted above, a high μ makes the attraction motive relatively more important and pushes firms to reveal more information. This increases the likelihood that the consumer opts for the outside option eventually, thus diminishing industry profit. We show that the second effect can be so strong that industry profit may be decreasing in the average product quality.

Corollary 6.4. *(i) Suppose $u_0 < \mu^{FD} - c$. There exists a $\mu^* < \mu^{FD}$ such that a firm's equilibrium profit is increasing in μ for all $\mu < \mu^*$, and decreasing in μ for $\mu \in \left(\mu^*, \mu^{FD}\right)$.*

[22] Choi et al. (2018) report a similar finding in their price-competition setting.

(ii) A firm's equilibrium profit is weakly decreasing in the consumer's search cost c, and strictly so if u_0 is in some intermediate region.

7 Discussion and Concluding Remarks

In this paper, we explore competition in information provision in a sequential, directed search setting. By developing a geometric approach to the optimal information design problem in this environment, we are able to characterize the unique symmetric equilibrium via the simple linear structure of the payoff function it induces. We illustrate the power of this technique beyond the symmetric setting with a preliminary investigation of asymmetric equilibria and competition between heterogeneous firms. We believe our approach may prove useful in other applications of competitive information design in which search frictions arise naturally.

Our model highlights the key economic forces at work in this class of environments; namely, the conflicting motives of attraction and persuasion. We uncover a number of insights pertaining to how the underlying environment shapes these two incentives and the ensuing information provision. For instance, a sufficiently high average quality and/or a large number of competitors makes the attraction motive dominant, leading to full information in equilibrium.[23] More generally, an improvement in average product quality can be detrimental to firms' equilibrium profits because the cost of excessive information revelation outweighs its direct benefit.

Outside of the high average quality case, although the attraction incentive remains, the persuasion motive has more of an effect: if everyone else provides full information, it is now worthwhile for a firm to provide no information and count on the consumer to visit and select it at the end of a (theretofore unsuccessful) search. The forces of attraction and persuasion can be balanced only in a mixed-strategy equilibrium, resulting in dispersion in information provision (despite the market's *ex-ante* homogeneity). By contrasting this finding with alternative settings

[23] This result rings true, in the sense that there are many markets in which firms seem to provide a large amount of information to visiting consumers. The fine piano purveyor, Steinway & Sons, has practice rooms in its showrooms so that people can get a feel for the instrument themselves, many car dealerships allow test-drives and some even allow prospective buyers to keep the car overnight, upscale clothing stores include changing rooms (and mirrors) for trying on their products, and anyone who has mistakenly wandered through the perfume section of a department store can attest that the perfume sellers provide ample olfactory evidence about their wares.

in which the attraction motive is irrelevant, we show not only that the attraction motive is key to generating the information dispersion result, but also, somewhat counterintuitively, that the consumer can actually benefit from having a small positive search cost (rather than none).

The assumption that firms have perfect flexibility in their ability to design signals not only aids us in obtaining a straightforward equilibrium characterization but also helps make transparent the aforementioned tension between attraction and persuasion. While reducing noise always helps attract the consumer, the effectiveness of persuasion can be enhanced only by introducing noise in a specific manner.[24] A restrictive set of feasible signals might thus obscure our model's essential trade-off. Moreover, although in practice, a firm's control over information revelation is never perfect, the economic insights we uncover remain valid provided the set of feasible signals is not too meagre. Consider, for instance, the result of full disclosure with a sufficiently high μ or n. If the signal space were more restrictive than the one we consider, the attraction motive would still dominate if competition were sufficiently intense. In sum, the general approach that we take in this paper, in which firms have the flexibility to design any signal, allows us to cleanly illustrate the fundamental forces at work.

Our analysis provides a number of testable predictions that await empirical investigation. Indeed, one of the main ideas emerging from our analysis is the possibility of information dispersion when competition is not that intense. Do we observe such information dispersion in real-world markets? Unlike price dispersion, which is relatively easy to observe and measure (since prices are merely scalars), information levels are much harder to quantify, which could explain the dearth of formal evidence of this phenomenon. This difficulty notwithstanding, casual observation suggests that such variation does exist. An alternative interpretation of our model is that firms compete by choosing their product designs, which affect the distributions of their match values with the consumers. Naturally, a broad design induces a more concentrated distribution of match values, whereas a niche design induces a more spread-out distribution. With this interpretation, our model suggests that design dispersion can arise despite *ex-ante* homogeneity.

[24] Indeed, how to do this optimally is the central theme of the literature on Bayesian persuasion.

A Sections 3, 4, and 5 Proofs

A.1 Proof of Lemma 3.2

This is a special case of Lemma 4.1 and is thus omitted.

A.2 Proof of Proposition 3.3

This proposition is a special case of Proposition 4.5; specifically, case (iii) of Lemma 4.3.

A.3 Proof of Lemma 3.6

Given a distribution F over posteriors, the mean of the effective-value distribution it implies is computed as follows:

$$\begin{aligned}\int_0^{\bar{U}} w dH_F(w) &= \bar{U} - \int_0^{\bar{U}} H_F(w)\,dw = \bar{U} - \int_0^{U} F(w)\,dw - \int_U^{\bar{U}} 1dw \\ &= U - (U + c - \mu) = \underline{U}.\end{aligned}$$

Here, the first equality makes use of integration by parts, the second equality makes use of the definition of H_F (given by (2)), and the third equality makes use of the reservation-value equation (1), which implies $\int_0^U F(w)\,dw = U + c - \mu$.

Using the reservation-value equation (1) again, the expected effective value conditional on falling short of U is given by

$$\frac{\int_0^U w dF(w)}{F(U)} = U - \frac{U - \underline{U}}{F(U)}. \tag{13}$$

It is clearly increasing in $F(U)$. As $\int_U^1 F(w)\,dw = 1 - c - U$, the value of $F(U)$ is maximized if F is flat over the interval $(U, 1)$, in which case $F(U) = 1 - \frac{c}{1-U}$. In a similar vein, using the implication of the reservation-value equation that $\int_0^U F(w)\,dw = U - \underline{U}$, the value of $F(U)$ is minimized if F is flat over $(0, U)$, in which case $F(U) = 1 - \frac{\underline{U}}{U}$. Substituting these bounds on $F(U)$ into (13) yields the bounds on the expected effective value conditional on falling short of U as stated in the lemma.

A.4 Proof of Lemma 4.1

Suppose H has an atom at some $\tilde{w} \notin \left\{0, \bar{U}\right\}$. We show that the best response to $\Pi(\cdot; H)$ does not put any positive mass at $\tilde{w}$. Using Proposition 3.7, the best response to $\Pi(\cdot; H)$ puts a positive mass at $\tilde{w}$ only if either there is some $U > \tilde{w}$ such that $\Pi_U(\tilde{w}; H) = \hat{\Pi}_U(\tilde{w}; H)$, or if $\tilde{w}$ is an optimal reservation value. First, as H has an atom at $\tilde{w}$, we have $\Pi(\tilde{w}; H) < \lim_{w \to \tilde{w}^+} \Pi(w; H)$. Consequently, $\Pi_U(\tilde{w}; H) < \hat{\Pi}_U(\tilde{w}; H)$ for all $U > \tilde{w}$. Moreover, $\tilde{w}$ cannot possibly be an optimal reservation value either. To see this, recall the maximal payoff given reservation value $\tilde{w}$ is

$$V(\tilde{w}) = \max_{a \in [0, \bar{a}(\tilde{w})]} \Pi(\tilde{w}; H) - \frac{\Pi(\tilde{w}; H) - \hat{\Pi}_{\tilde{w}}(a; H)}{\tilde{w} - a} [\tilde{w} - \underline{U}].$$

Take an $\bar{\varepsilon} > 0$ sufficiently small so that $\Pi(w; H)$ is continuous over $(\tilde{w}, \tilde{w} + \bar{\varepsilon})$. Now for $\varepsilon \in (0, \bar{\varepsilon})$,

$$\begin{aligned} V(\tilde{w} + \varepsilon) &= \max_{a \in [0, \bar{a}(\tilde{w}+\varepsilon)]} \Pi(\tilde{w} + \varepsilon; H) - \frac{\Pi(\tilde{w} + \varepsilon; H) - \hat{\Pi}_{\tilde{w}+\varepsilon}(a; H)}{\tilde{w} + \varepsilon - a} [\tilde{w} + \varepsilon - \underline{U}] \\ &\geq \max_{a \in [0, \bar{a}(\tilde{w}+\varepsilon)]} \lim_{w \to \tilde{w}^+} \Pi(w; H) - \frac{\lim_{w \to \tilde{w}^+} \Pi(w; H) - \hat{\Pi}_{\tilde{w}+\varepsilon}(a; H)}{\tilde{w} + \varepsilon - a} [\tilde{w} + \varepsilon - \underline{U}], \end{aligned}$$

as Π is nondecreasing. By the theorem of the maximum, the last expression above is continuous in ε.[25] Taking limits on both sides yields

$$\lim_{\varepsilon \to 0} V(\tilde{w} + \varepsilon) \geq \max_{a \in [0, \bar{a}(\tilde{w})]} \lim_{w \to \tilde{w}^+} \Pi(w; H) - \frac{\lim_{w \to \tilde{w}^+} \Pi(w; H) - \hat{\Pi}_{\tilde{w}}(a; H)}{\tilde{w} - a} [\tilde{w} - \underline{U}],$$

which strictly exceeds $V(\tilde{w})$ since $\lim_{w \to \tilde{w}^+} \Pi(w; H) > \Pi(\tilde{w}; H)$.

A.5 Proof of Lemma 4.2

The expected payoffs at effective values $w = 0$, $\hat{U}$, and $\bar{U}$ stated in (7) follow from the definitions of α and $\hat{U}$. Therefore, it suffices to show that $\Pi(w; H)$ is necessarily linear over $w \in \left(0, \hat{U}\right]$.

First, in equilibrium, the restriction of payoff function Π to the domain $(0, \underline{U}]$ must be weakly concave, i.e., $\Pi_{\underline{U}}(w; H) = \hat{\Pi}_{\underline{U}}(w; H)$ for all $w \in (0, \underline{U}]$. Suppose to the contrary that there is some $w' \in (0, \underline{U})$ such that $\Pi_{\underline{U}}(w'; H) < \hat{\Pi}_{\underline{U}}(w'; H)$. Then there must be an open neighbourhood around w' over which no optimal effective-value distribution assigns any positive measure, and $\Pi_{\underline{U}}(\cdot; H)$ must be flat over this neighbourhood. As $\Pi_{\underline{U}}(w'; H) < \hat{\Pi}_{\underline{U}}(w'; H)$ implies that there is

[25] Note that $\hat{\Pi}_{\tilde{w}+\varepsilon}(a; H)$ is concave and thus continuous in a.

some $w_1 \in (w', \underline{U})$ such that $\Pi_{\underline{U}}(w'; H) < \Pi_{\underline{U}}(w_1; H)$, $\Pi_{\underline{U}}$ must have a discrete jump (and hence H has an atom) somewhere in the interval $[w', w_1]$, contradicting Lemma 4.1.

Next, we show that $\Pi_{\underline{U}}(w; H) = \hat{\Pi}_{\underline{U}}(w; H)$ must be linear over the region $(0, \underline{U}]$. Note that the equilibrium payoff can always be achieved by no disclosure, so $\Pi(\underline{U}; H) = 1/n$; otherwise, an atom must be found at $\inf(supp(H)) \cap \left[\underline{U}, \bar{U}\right]$, contradicting Lemma 4.1. The linearity of $\Pi_{\underline{U}}$ may therefore be stated as $\Pi_{\underline{U}}(w; H) = (\alpha(1-\mu))^{n-1} + \frac{n^{-1}-(\alpha(1-\mu))^{n-1}}{\underline{U}} w$ for all $w \in (0, \underline{U}]$. The weak concavity of $\Pi_{\underline{U}}$ implies that if it is not linear, then $\Pi_{\underline{U}}(w'; H) > (\alpha(1-\mu))^{n-1} + \frac{n^{-1}-(\alpha(1-\mu))^{n-1}}{\underline{U}} w'$ for all $w' \in (0, \underline{U})$. Using the concavification approach of effective-value optimization, effective values arbitrarily close to 0 can therefore lie on the support of H if and only if effective values arbitrarily close to $\bar{U}$ are also on the support, i.e., $\hat{U} = \bar{U}$. The expected payoff of offering a reservation value arbitrarily close to $\hat{U}$ converges to

$$\frac{\underline{U}}{\hat{U}}\Pi\left(\hat{U}; H\right) + \left(1 - \frac{\underline{U}}{\hat{U}}\right)\Pi\left(a\left(\hat{U}\right); H\right) \longrightarrow \frac{\underline{U}}{\bar{U}}(1-\alpha\mu)^{n-1} + \left(1 - \frac{\underline{U}}{\bar{U}}\right)(\alpha(1-\mu))^{n-1}.$$

As the payoff of offering reservation value $\hat{U}$ must also equal $1/n$, we have

$$\frac{1}{n} = \mu(1-\alpha\mu)^{n-1} + (1-\mu)(\alpha(1-\mu))^{n-1}.$$

The expected payoff of offering reservation value $\bar{U}$ is given by

$$\frac{1}{n} = \mu\frac{1-(1-\alpha\mu)^{n}}{n\alpha\mu} + (1-\mu)\frac{(\alpha(1-\mu))^{n-1}}{n}.$$

The only case where both of the equations above hold is when $\alpha = 0$ and $\mu = 1/n$.

Claim A.1. *Suppose Π is such that $\Pi_{\underline{U}}$ is concave and non-linear. Suppose also that $\Pi(\underline{U})$ gives the maximum payoff under Π. There exists an $\varepsilon > 0$ such that for all reservation values $U \in \left[\hat{U} - \varepsilon, \hat{U}\right]$ that can bring about the optimal payoff $\Pi(\underline{U})$, the corresponding effective-value distributions must have support $\{\bar{a}(U), U\}$.*

Proof. Observe first that $\Pi(U) < \Pi(\underline{U}) + \frac{\Pi(\hat{U})-\Pi(\underline{U})}{\hat{U}-\underline{U}} \times (U - \underline{U})$ for all $U \in \left[\underline{U}, \hat{U}\right]$; otherwise, the assumption of $\Pi_{\underline{U}}$ being concave and non-linear ($\Pi_{\underline{U}}(w'; H) > \frac{\Pi(\underline{U})}{\underline{U}} \times w'$ for all $w' \in (0, \underline{U})$) would imply that $\underline{U}$ is suboptimal.

Next, we establish that if reservation value U can deliver this the optimal payoff $\Pi(\underline{U})$, it can be achieved by an effective-value distribution with binary support $\{w, U\}$ for some $w < \bar{a}(U)$.

On the graph of Π, $(\underline{U}, \Pi(\underline{U}))$ lies on the straight line connecting $(U, \Pi(U))$ and $\left(w, \hat{\Pi}_U(w)\right)$ for some $w < \bar{a}(U)$. If $\Pi(w) < \hat{\Pi}_U(w)$, there must be some $u' \in (\underline{U}, U)$ such that $\left(w, \hat{\Pi}_U(w)\right)$ can be expressed as a convex combination of $(u', \Pi(u'))$ and $(w', \Pi(w'))$, for some $w' < w$. This is, however, impossible, as this implies that a feasible effective-value distribution (w', u') can achieve a payoff exceeding $\Pi(\underline{U})$. Therefore, we must have $\Pi(w) = \hat{\Pi}_U(w)$.

Now as reservation value U is optimal and $\Pi_{\underline{U}}$ is concave, either $w = \bar{a}(U)$ or Π is linear over $[w, \underline{U}]$. If Π is not linear over $[w, \underline{U}]$, we are done as this implies that for all optimal reservation values U' larger than U , the corresponding effective-value distributions must have support $\{\bar{a}(U'), U'\}$. However, as Π is not fully linear and $\Pi_{\underline{U}}$ is concave, reservation values sufficiently close to $\hat{U}$ must achieve the optimal payoff by an effective-value distribution with support $\{\bar{a}(U), U\}$. This completes the proof of the claim. ∎

Suppose $\Pi_{\underline{U}}$ is indeed nonlinear in equilibrium and that $\hat{U} = \bar{U}$. The claim above pins down the form of the effective-value distributions for reservation values U close to $\bar{U}$: they must all have support $\{\bar{a}(U), U\}$. With this restriction, the payoff of a firm offering reservation value $U \in \left[\bar{U} - \varepsilon, \bar{U}\right]$ can be expressed as

$$\Psi(U) = \left(1 - \frac{c}{1-U}\right)\left(1 - G(U) - c\int_U^{\bar{U}} \frac{1}{1-s} dG(s)\right)^{n-1} + \frac{c}{1-U}\left(1 - c\int_U^{\bar{U}} \frac{1}{1-s} dG(s)\right)^{n-1},$$

where $G(\cdot)$ is the distribution of reservation values. As all reservation values $U \in \left[\bar{U} - \varepsilon, \bar{U}\right]$ lead to the same payoff, $\Psi'(U) = 0$ or

$$\begin{aligned} 0 \quad = \quad & \frac{c}{(1-U)^2}\left[\left(1 - c\int_U^{\bar{U}} \frac{1}{1-s} dG(s)\right)^{n-1} - \left(1 - G(U) - c\int_U^{\bar{U}} \frac{1}{1-s} dG(s)\right)^{n-1}\right] \\ & + (n-1)\left[\left(\frac{c}{1-U}\right)^2\left(1 - c\int_U^{\bar{U}} \frac{1}{1-s} dG(s)\right)^{n-2}\right. \\ & \left. - \left(1 - \frac{c}{1-U}\right)^2\left(1 - G(U) - c\int_U^{\bar{U}} \frac{1}{1-s} dG(s)\right)^{n-2}\right]\frac{dG(U)}{dU}, \end{aligned}$$

provided that $n \geq 3$. Substituting $U = \bar{U}$ gives

$$\Psi'\left(\bar{U}\right) = 0 \iff 0 = \frac{c}{\left(1-\bar{U}\right)^2} + (n-1)\left(\frac{c}{1-\bar{U}}\right)^2 \frac{dG\left(\bar{U}\right)}{dU},$$

which is impossible. If $n = 2$, $\Psi'(U) = 0$ simplifies to $\left(\frac{2c}{1-U} - 1\right) g(U) + \frac{c}{(1-U)^2} G(U) = 0$. Evidently, this equation is impossible at $U = \bar{U}$.

Finally, the linearity of the payoff function $\Pi(w; H)$ over $w \in \left(0, \hat{U}\right]$ follows immediately from the linearity of $\Pi_{\underline{U}}$. In fact, it is necessary that

$$\Pi(w; H) = (\alpha(1-\mu))^{n-1} + \frac{n^{-1} - (\alpha(1-\mu))^{n-1}}{\underline{U}} w,$$

for all $w \in \left(0, \hat{U}\right]$. If $\Pi(w'; H) > (\alpha(1-\mu))^{n-1} + \frac{n^{-1}-(\alpha(1-\mu))^{n-1}}{\underline{U}} w'$ for some $w' \in \left(\underline{U}, \hat{U}\right]$, then a firm can achieve an expected payoff exceeding $1/n$ by offering reservation value w', a contradiction. If $\Pi(w'; H) < (\alpha(1-\mu))^{n-1} + \frac{n^{-1}-(\alpha(1-\mu))^{n-1}}{\underline{U}} w'$ for some $w' \in \left(\underline{U}, \hat{U}\right]$, this reservation value is sub-optimal and there is an open neighbourhood around it that does not lie on the support of H. This implies an atom somewhere in $\left(w', \hat{U}\right)$, again a contradiction.

A.6 Proof of Lemma 4.3

Substituting (7) into equations (8) and (9) and simplifying gives, respectively,

$$\hat{U} = \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{n^{-1} - (\alpha(1-\mu))^{n-1}} \underline{U}, \text{ and} \tag{14}$$

$$(1-\alpha\mu)^n - (\alpha(1-\mu))^n = 1 - \alpha \tag{15}$$

We begin by showing that (15) has a unique solution in $\alpha \in (0,1)$ if and only if $\mu \in \left(n^{-1}, 1 - n^{-\frac{1}{n-1}}\right)$. To this end, define $T : [0,1] \longrightarrow \mathbb{R}$ by $T(\alpha) \equiv (1-\alpha\mu)^n - (\alpha(1-\mu))^n - (1-\alpha)$. The following observations are immediate but useful. First, 0 and 1 are both roots of T. Second, by direct computation, $T''(\alpha) > 0 \iff \alpha < \left(\mu + \left(\mu^{-2}(1-\mu)^n\right)^{\frac{1}{n-2}}\right)^{-1}$, so T'' changes sign at most once. Third, $T'(0) = 1 - n\mu$ and $T'(1) = 1 - n(1-\mu)^{n-1}$.

The case $\mu \leq n^{-1}$ has $T'(0) \geq 0$ and $T'(1) < 0$. The fact that T'' changes sign only once implies that T' also changes sign only once. As $T(0) = T(1) = 0$, it is necessary that $T(a) > 0$ for all $\alpha \in (0,1)$.

The case $\mu \in \left(n^{-1}, 1 - n^{-\frac{1}{n-1}}\right)$ has $T'(0) < 0$ and $T'(1) < 0$. The fact that T'' changes sign only once, together with $T(0) = T(1) = 0$, implies that T' is positive if and only if α lies in some interior interval. Therefore, T crosses the horizontal axis once and only once, and it occurs in this interval.

The case $\mu \geq 1 - n^{-\frac{1}{n-1}}$ has $T'(0) < 0$ and $T'(1) \geq 0$. The fact that T'' changes sign only once implies that T' also changes sign only once. As $T(0) = T(1) = 0$, it is necessary that $T(a) < 0$ for all $\alpha \in (0,1)$.

Summing up the observations above reveals that (15) has a unique solution in $\alpha \in (0,1)$ if and only if $\mu \in \left(n^{-1}, 1 - n^{-\frac{1}{n-1}}\right)$.

Consider an equilibrium with $\alpha \in (0,1)$. We have shown above that this is feasible only if $\mu \in \left(n^{-1}, 1 - n^{-\frac{1}{n-1}}\right)$. It remains to show that in this case, $\hat{U}$ given by (14) lies between $\underline{U}$ and $\bar{U}$. It is immediate that $\hat{U} \geq \underline{U}$ is equivalent to $n^{-1} \leq (1 - \alpha\mu)^{n-1}$, which follows immediately from $\mu < 1 - n^{-\frac{1}{n-1}}$ and $\alpha \in (0,1)$. Straightforward algebra reveals that $\hat{U} \leq \bar{U}$ is equivalent to $(1 - \alpha\mu)^{n-1} \leq \alpha n^{-1} + (1 - \alpha)$. As α and μ are related by (15), the last inequality is equivalent to requiring that

$$S(\alpha) \equiv \left(\alpha n^{-1} + (1-\alpha)\right)^{\frac{n}{n-1}} - (1-\alpha) - \left(\left(\alpha n^{-1} + (1-\alpha)\right)^{\frac{1}{n-1}} - (1-\alpha)\right)^{n}$$

be non-negative for all $\alpha \in (0,1)$. To show this, denote $\beta(\alpha) \equiv \left(\alpha n^{-1} + (1-\alpha)\right)^{\frac{1}{n-1}}$, so that β has a range of $\left[n^{-\frac{1}{n-1}}, 1\right]$. As $\beta'(\alpha) = -\left(n\beta(\alpha)^{n-2}\right)^{-1}$, the derivative of S can then be expressed as

$$S'(\alpha) = 1 - \beta - n(\beta - 1 + \alpha)^{n-1}\left(1 - \left(n\beta^{n-2}\right)^{-1}\right).$$

It is straightforward to verify that $S(0) = S(1) = 0$ and $S'(0) = S'(1) = 0$. The claim can be shown by proving that there is a unique cutoff value such that $S'(\alpha) \geq 0$ if and only if α is below the cutoff. Using the formula for S' and the definition of β, $S' \geq 0$ if and only if

$$\frac{\beta^{n-1} - 1}{n^{-1} - 1} \leq \left(\frac{1-\beta}{n - \beta^{-(n-2)}}\right)^{\frac{1}{n-1}} + 1 - \beta. \tag{16}$$

Therefore, it suffices to show that there exists a $\hat{\beta} \in \left(n^{-\frac{1}{n-1}}, 1\right)$ such that (16) holds if and only if $\beta > \hat{\beta}$. Rearranging (16) gives

$$L(\beta) \equiv \left(\frac{\beta^{n-1} - 1}{n^{-1} - 1} + \beta - 1\right)^{n-1} \leq \frac{1-\beta}{n - \beta^{-(n-2)}} \equiv R(\beta)$$

Note that $L\left(n^{-\frac{1}{n-1}}\right) = R\left(n^{-\frac{1}{n-1}}\right)$ and $L(1) = R(1)$. Moreover, $L'(\beta), R'(\beta) < 0$. Also, $R''(\beta) > 0$; whereas $L''(\beta)$ switches sign only once, from negative to positive. Taken together, $L(\beta) \leq R(\beta)$ if and only if β is sufficiently large.

Consider next an equilibrium with $\alpha = 0$. (14) is necessary and gives $\hat{U} = n\underline{U}$. Clearly, this is feasible if and only if $\hat{U} \leq \bar{U} \Longleftrightarrow \mu \leq n^{-1}$.

Finally, consider an equilibrium with $\alpha = 1$. The requirement $\Pi(\underline{U}; H) \leq n^{-1}$ boils down to $(1-\mu)^{n-1} \leq n^{-1}$, which is equivalent to $\mu \geq 1 - n^{-\frac{1}{n-1}}$.

A.7 Proof of Lemma 4.4

Because of the continuity of $H(w)$ (given by (7)) over $\left(0,\hat{U}\right]$, it suffices to show that in this region, there is a mixed strategy that can match its density, denoted by h, which is given by

$$h(w)=\frac{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}{(n-1)\hat{U}}\left((\alpha(1-\mu))^{n-1}+\frac{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}{\hat{U}}w\right)^{-\frac{n-2}{n-1}}. \tag{17}$$

To this end, define a mapping $b:\left[\underline{U},\hat{U}\right]\longrightarrow[0,\underline{U}]$ by $K(U)=K(b(U))$, where $K:\left[0,\bar{U}\right]\longrightarrow\mathbb{R}$ is given by

$$\begin{aligned}K(w) \equiv & \left((\alpha(1-\mu))^{n-1}+\left((1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}\right)\frac{w}{\hat{U}}\right)^{\frac{1}{n-1}}\\ & \times\left(n\underline{U}+(n-1)\left(\frac{\hat{U}}{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}\right)(\alpha(1-\mu))^{n-1}-w\right),\end{aligned}$$

and parameters α and $\hat{U}$ are as given in Lemma 4.3. For each $U\in\left[\underline{U},\hat{U}\right]$, let F_U be a binary distribution with support $\{b(U),U\}$ and mean $\underline{U}$; and let $F_{\bar{U}}$ be the binary distribution with support $\left\{0,\bar{U}\right\}$ and mean $\underline{U}$. Moreover, let G be a reservation-value distribution that has an atom $\alpha\in[0,1]$ at $\bar{U}$ and a density for $U\in\left[\underline{U},\hat{U}\right]$ as follows:

$$g(U)\equiv\frac{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}{(n-1)\hat{U}}\left((\alpha(1-\mu))^{n-1}+\frac{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}{\hat{U}}U\right)^{-\frac{n-2}{n-1}}\frac{U-b(U)}{\underline{U}-b(U)}.$$

Below, we show that the mixed strategy $\left(G,\{F_U(\cdot)\}_{U\in\left[\underline{U},\hat{U}\right]\cup\left\{\bar{U}\right\}}\right)$ generates the effective-value distribution H (defined in (12)).

First, we show that effective-value distribution F_U is inducible for each $U\in\left[\underline{U},\hat{U}\right]\cup\left\{\bar{U}\right\}$. Note that the mapping b is well-defined: a direct computation reveals that $K(w)$ is strictly concave with a peak at $\underline{U}$ and that $K(0)=K\left(\hat{U}\right)$. We need to show that $b(U)\le\bar{a}(U)\equiv\frac{\mu-c-\mu U}{1-c-U}$ (as defined in Lemma 3.6). Furthermore, note that because $\bar{a}(\underline{U})=b(\underline{U})$, $b\left(\hat{U}\right)=0=\bar{a}\left(\bar{U}\right)\le\bar{a}\left(\hat{U}\right)$, and $\bar{a}(U)$ is decreasing and strictly concave, it suffices to show that $b(U)$ is convex. To this end, we adopt a change of variable: let $v=U-\underline{U}$, and $d(v)=\underline{U}-b(\underline{U}+v)$.[26] The implicit definition of b implies $K(\underline{U}+v)=K(\underline{U}-d(v))$; or equivalently,

$$(M+v)^{\frac{1}{n-1}}((n-1)M-v)=(M-d(v))^{\frac{1}{n-1}}((n-1)M+d(v)), \tag{18}$$

[26] Intuitively, $d(\cdot)$ represents the "reflection" of U about $\underline{U}$ according to the function K.

where $M \equiv \underline{U} + \frac{\hat{U}(\alpha(1-\mu))^{n-1}}{(1-\alpha\mu)^{n-1}-(\alpha(1-\mu))^{n-1}}$. It follows that $d(v) < v$.[27] Now, $b(U)$ is convex if and only if $d''(v) \leq 0$. Using (18), $d''(v) \leq 0$ holds if and only if

$$\frac{K''\left(\underline{U} - d(v)\right)}{\left(K'\left(\underline{U} - d(v)\right)\right)^2} \geq \frac{K''\left(\underline{U} + v\right)}{\left(K'\left(\underline{U} + v\right)\right)^2}.$$

Direct computation of the derivatives shows that the inequality above holds if and only if

$$\left(\frac{M - d(v)}{M + v}\right)^{\frac{1}{n-1}} \leq \left(\frac{M(n-1) - d(v)}{(d(v))^2}\right) / \left(\frac{M(n-1) + v}{v^2}\right).$$

Using (18) again, the inequality above is equivalent to requiring $d(v) < v$.

It remains to verify that the effective-value density implied by the mixed strategy defined above matches with $h(w)$ (given in (17)). Consider the effective values above $\underline{U}$. Note that these effective values are realized as reservation values in the mixed strategy $\left(G, \{F_U(\cdot)\}_{U \in [\underline{U}, \hat{U}] \cup \{\bar{U}\}}\right)$. For $w \geq \underline{U}$, the density implied by the mixed strategy is

$$\begin{aligned}
& g(w) \times \frac{\underline{U} - b(w)}{w - b(w)} \\
= \; & \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\hat{U}} \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U}} w\right)^{-\frac{n-2}{n-1}} \frac{w - b(w)}{\underline{U} - b(w)} \times \frac{\underline{U} - b(w)}{w - b(w)} \\
= \; & h(w).
\end{aligned}$$

Now consider the effective values below $\underline{U}$. These effective values are realized as bad posterior realizations in the mixed strategy. Define by $q : [0, \underline{U}] \longrightarrow \left[\underline{U}, \hat{U}\right]$ the inverse of mapping b. For $w \leq \underline{U}$, the density implied by the mixed strategy is

$$\begin{aligned}
& -q'(w) \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \; & -\frac{K'(w)}{K'(q(w))} \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \; & \frac{\left((\alpha(1-\mu))^{n-1} + \left((1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}\right) \frac{w}{\hat{U}}\right)^{-\frac{n-2}{n-1}}}{\left((\alpha(1-\mu))^{n-1} + \left((1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}\right) \frac{q(w)}{\hat{U}}\right)^{-\frac{n-2}{n-1}}} \times \frac{\underline{U} - w}{q(w) - w} \times g(q(w)) \\
= \; & h(w),
\end{aligned}$$

where the first equality makes use of the definition of the mapping q, and the second and last equalities make use of the definitions of the functions K and g, respectively.

[27] To see this precisely, note that the function $\left[(M+v)^{\frac{1}{n-1}}((n-1)M - v)\right] - \left[(M-v)^{\frac{1}{n-1}}((n-1)M + v)\right]$ is equal to 0 when $v = 0$ and is increasing in v for all $v \geq 0$. Moreover, $(M-d)^{\frac{1}{n-1}}((n-1)M + d)$ is decreasing in d for all $d \geq 0$.

A.8 Proof of Corollary 4.6

The case of $\mu > \bar{\mu}$ is trivial since the distributions are binary with support $\{0, 1 - c/\mu\}$. Consider next the case $\mu < \bar{\mu}$. As shown in Lemma 4.3, α is independent of c, so it suffices to check the continuous portion of cdf H and $\hat{U}$. The latter is linear and decreasing in c, whereas the continuous portion of H is

$$H(w) = \left((\alpha(1-\mu))^{n-1} + \frac{1 - n(\alpha(1-\mu))^{n-1}}{n(\mu - c)} w\right)^{\frac{1}{n-1}},$$

which is obviously increasing in c. Finally, according to Corollary 1 of Choi et al. (2018), and appealing to the Law of Iterated Expectations (since we are evaluating welfare from an *ex-ante* point of view), the consumer's *ex-ante* payoff is given by the expectation of the highest effective value. A worse effective-value distribution in the sense of first-order stochastic dominance thus lowers the consumer's *ex-ante* payoff.

A.9 Proof of Proposition 5.1

For $c = 0$, the effective value is simply the posterior, as any feasible distribution over posteriors has a reservation value equal to one. The symmetric equilibrium distribution H_0 of effective values is computed in Au and Kawai (2020) and takes the form:

$$H_0(w) = \begin{cases} (1 - \alpha_0\mu) \times \left(\frac{w}{\hat{U}_0}\right)^{\frac{1}{n-1}} & \text{if } w \in \left[0, \hat{U}_0\right] \\ 1 - \alpha_0\mu & \text{if } w \in \left(\hat{U}_0, 1\right) \\ 1 & \text{if } w = 1 \end{cases}.$$

By aligning the slopes of the implied payoff function ($\Pi(1; H_0) = \frac{\Pi(\hat{U}_0; H_0)}{\hat{U}_0}$ if $\alpha_0 > 0$) and the Bayes-plausibility condition ($\int_0^1 w dH_0(w) = \mu$), the α_0 and $\hat{U}$ can be pinned down as follows. If $\mu \leq \underline{\mu} = n^{-1}$, $\alpha_0 = 0$ and $\hat{U} = n\mu$. If $\mu > \underline{\mu}$, then $\hat{U}_0 = \frac{n\alpha_0\mu(1-\alpha_0\mu)^{n-1}}{1-(1-\alpha_0\mu)^n}$, where α_0 is the unique solution to $(1 - \alpha_0\mu)^n = 1 - \alpha_0$.

For the case $\mu \leq \underline{\mu}$, the convergence of H_c to H^* is immediate by comparing the distribution reported in Lemma 4.3 with H_0 stated above.

Next, consider the case $\mu \in \left(\underline{\mu}, \bar{\mu}\right)$. With $c \longrightarrow 0$, $\underline{U} \longrightarrow \mu$; and hence H^* is given by (12) with $(1 - \alpha\mu)^n - (\alpha(1-\mu))^n = 1 - \alpha$, and $\hat{U} = \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{n^{-1} - (\alpha(1-\mu))^{n-1}}\mu$. Recall from the proof of Lemma

4.3 that when $\mu \in (\underline{\mu}, \bar{\mu})$, $T(\tilde{\alpha}) \equiv (1-\tilde{\alpha}\mu)^n - (\tilde{\alpha}(1-\mu))^n - (1-\tilde{\alpha})$ is negative if and only if $\tilde{\alpha}$ is less than the interior root α of T. As $T(\alpha_0) < 0$, it follows that $\alpha_0 < \alpha$. Now, $\Pi(w; H_0)$ is fully linear over $\left(0, \hat{U}_0\right)$, has a zero vertical intercept, and passes through the points $\left(\mu, n^{-1}\right)$ and $\left(\hat{U}_0, (1-\alpha_0\mu)^{n-1}\right)$; whereas $\Pi(w; H^*)$ is linear over $\left(0, \hat{U}\right)$, has a positive vertical intercept $((\alpha(1-\mu))^{n-1})$, and passes through the points $\left(\mu, n^{-1}\right)$ and $\left(\hat{U}_0, (1-\alpha\mu)^{n-1}\right)$. Therefore, $\Pi(w; H_0)$ and $\Pi(w; H^*)$ have a unique intersection at μ over interior effective values $(0,1)$. As H_0 and H^* have no interior atoms, $\Pi(w; H_0) = H_0(w)^{n-1}$ and $\Pi(w; H^*) = H^*(w)^{n-1}$ for $w \in (0,1)$. As a result, H_0 and H^* have a unique interior intersection at μ, at which H_0 cuts H^* from below. As both H_0 and H^* have the same mean of μ, it follows that H^* is a mean-preserving spread of H_0.

The case of $\mu \geq \bar{\mu}$ is immediate: H^* corresponds to full disclosure, whereas H_0 is strictly partial.

A.10 Proof of Proposition 6.1

Because the game is zero-sum, the first part of this result is trivial. If there existed an asymmetric equilibrium then there would exist multiple pure strategy equilibria. We have already established the uniqueness of such an equilibrium so by contraposition the result is shown.

To establish the second part of the proposition, observe that the corresponding distribution over values played by the $n-1$ firms is just the prior and is induced by providing full information. For each of these firms, its payoff as a function of the induced effective value, w, is

$$\Pi(w) = \begin{cases} \frac{(1-\mu)^{n-2}}{2(\mu-c)} w & \text{if } w \in [0, 2(\mu-c)] \\ (1-\mu)^{n-2} & \text{if } w \in \left(2(\mu-c), \bar{U}\right) \\ \frac{1-(1-\mu)^{n-1}}{\mu(n-1)} & \text{if } w = \bar{U} \end{cases}.$$

Thus, the optimal distribution is either the Bernoulli distribution (full information), yielding a payoff of $\left(1-(1-\mu)^{n-1}\right)/(n-1)$; or has support on $[0, 2(\mu-c)]$, yielding a payoff of $(1-\mu)^{n-2}/2$. Hence, we need

$$\frac{1-(1-\mu)^{n-1}}{n-1} \geq \frac{(1-\mu)^{n-2}}{2} \quad \Longleftrightarrow \quad \frac{2(1-\mu)}{n+1-2\mu} \geq (1-\mu)^{n-1}.$$

The construction of the effective-value distribution for the firm that is choosing H is described earlier in the paper (it is just the low mean two firm distribution and can be done, e.g., by mixing over binary distributions over posteriors). From any distribution over effective values, other than that corresponding to full information (the Bernoulli distribution over posteriors), firm n's payoff is $(1-\mu)^{n-1}$; whereas its payoff from full information is just $1/n$, since the vector of strategies would then be symmetric. Thus, the optimal distribution is either the Bernoulli distribution, yielding a payoff of $1/n$; or any distribution with support on $[0, 1-c/\mu]$, yielding a payoff of $(1-\mu)^{n-1}$. Accordingly, we need $(1-\mu)^{n-1} \geq 1/n$. Combining both conditions, we get

$$\frac{2(1-\mu)}{n+1-2\mu} \geq (1-\mu)^{n-1} \geq \frac{1}{n}.$$

B Section 6 Proofs

B.1 Proof of Theorem 6.2

The theorem compiles the results from the following four lemmata. One-by-one,

Lemma B.1. *If $\mu_1 - c \geq 1 - c/\mu_2$, there is a collection of equilibria in which firm 1 chooses the degenerate distribution over effective values, $\mu_1 - c$ with probability 1, and firm 2 chooses any distribution over effective values. Firm 1's distribution over effective values corresponds, e.g., to a completely uninformative signal.*

Proof. The result is trivial. Firm 1 is visited first and selected with certainty. ■

Lemma B.2. *If $\mu_2 \leq 1/2$ and $1 - c/\mu_2 \geq 2(\mu_1 - c)$, there is an equilibrium in which firm 1 and firm 2 choose distributions over effective values $H_1(w)$ and $H_2(w)$, respectively, where*

$$H_1(w) = \frac{w}{2(\mu_1 - c)}, \quad on \quad [0, 2(\mu_1 - c)],$$

and

$$H_2(w) = 1 - \frac{\mu_2 - c}{\mu_1 - c} + \left(\frac{\mu_2 - c}{\mu_1 - c}\right)\frac{w}{2(\mu_1 - c)}, \quad on \quad [0, 2(\mu_1 - c)].$$

Proof. It is easy to verify that these distributions are feasible, but remains to verify that they are inducible. To that end, we construct them as follows.

Firm 1's random (reservation) value U_1 is distributed according to distribution G_1:

$$G_1(u) \equiv \mathbb{P}(U_1 \leq u) = \frac{1}{\mu_1 - c}u - 1, \quad \text{on} \quad [\mu_1 - c, 2(\mu_1 - c)],$$

where for each $u \in [\mu_1 - c, 2(\mu_1 - c)]$, the distribution over posteriors, F_u, is the binary distribution with support $\{2(\mu_1 - c) - u, u + 2c\}$. In turn, Firm 2's random (reservation) value U_2 is distributed according to distribution G_2.

$$G_2(u) \equiv \mathbb{P}(U_2 \leq u) = \frac{1}{\mu_1 - c} u - 1, \quad \text{on} \quad [\mu_1 - c, 2(\mu_1 - c)],$$

where for each $u \in [\mu_1 - c, 2(\mu_1 - c)]$, F_u is given by the *ternary* distribution

$$F_u = \begin{Bmatrix} 0 & b(u) & a(u) \\ 1 - \frac{\mu_2 - c}{\mu_1 - c} & \frac{\mu_2 - c}{2(\mu_1 - c)} & \frac{\mu_2 - c}{2(\mu_1 - c)} \end{Bmatrix}, \quad \text{where} \quad a(u) \equiv u + 2c\frac{\mu_1 - c}{\mu_2 - c} \quad \text{and} \quad b(u) \equiv 2(\mu_1 - c) - u.$$

Recall that the top row of the matrix is the support of the distribution and the bottom row the associated probability weights (the pmf). Evidently, these constructions yield the desired distributions over effective values.

■

Lemma B.3. *If $\mu_2 \leq 1/2$ and $2(\mu_1 - c) > 1 - c/\mu_2 > \mu_1 - c$, there is an equilibrium in which firm 1 and firm 2 choose distributions over effective values H_1 and H_2, respectively, where*

$$H_1(w) = 2\left(1 - \frac{\mu_2(\mu_1 - c)}{\mu_2 - c}\right)\frac{\mu_2}{\mu_2 - c} w, \quad \textit{on} \quad \left[0, \frac{\mu_2 - c}{\mu_2}\right],$$

and

$$H_2(w) = 1 - 2\mu_2 + 2\mu_2 \frac{\mu_2}{\mu_2 - c} w, \quad \textit{on} \quad \left[0, \frac{\mu_2 - c}{\mu_2}\right].$$

Proof. As above, it is easy to verify that these distributions are feasible, but remains to verify that they are inducible. To that end, we construct them as follows.

Firm 1's random (reservation) value U_1, is distributed according to distribution G_1:

$$G_1(u) \equiv \mathbb{P}(U_1 \leq u) = 4\frac{\mu_2}{\mu_2 - c}\left(1 - \frac{\mu_2(\mu_1 - c)}{\mu_2 - c}\right)(u - (\mu_1 - c)), \quad \text{on} \quad \left[\mu_1 - c, \frac{\mu_2 - c}{\mu_2}\right],$$

where for each $u \in [\mu_1 - c, 1 - c/\mu_2)$, F_u is binary with support $\{b(u), a(u)\}$, where $a(u) \equiv u + 2c$ and $b(u) \equiv 2(\mu_1 - c) - u$; and $F_{\frac{\mu_2 - c}{\mu_2}}(x)$ is defined as

$$F_{\frac{\mu_2 - c}{\mu_2}}(x) = \begin{cases} \dfrac{2\left(\mu_1 - c - \frac{\mu_2 - c}{\mu_2}\right)}{3\left(\frac{\mu_2 - c}{\mu_2}\right)^2 - 8(\mu_1 - c)\frac{\mu_2 - c}{\mu_2} + 4(\mu_1 - c)^2} x & \text{if} \quad x \in \left[0, 2(\mu_1 - c) - \frac{\mu_2 - c}{\mu_2}\right] \\ 1 - \left(\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c\right) & \text{if} \quad x \in \left[2(\mu_1 - c) - \frac{\mu_2 - c}{\mu_2}, \frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c\right) \\ 1, & \text{if} \quad x \in \left[\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c, 1\right] \end{cases}.$$

Viz., $F_{\frac{\mu_2-c}{\mu_2}}(x)$ has a point mass of size

$$\frac{\frac{\mu_2-c}{\mu_2}}{3\frac{\mu_2-c}{\mu_2}-2(\mu_1-c)} \quad \text{on} \quad \frac{\mu_2-c}{\mu_2}+3c-\frac{2(\mu_1-c)\mu_2}{\mu_2-c}c.$$

Evidently, a is increasing in u and takes values in the interval $[\mu_1+c, 1-c/\mu_2+2c]$; and b is decreasing in u and takes values in the interval $[\mu_1-c, 2(\mu_1-c)-1+c/\mu_2]$. We should verify four things:

Claim B.4. *The upper bound of $a(u)$ is less than* 1, *i.e.*, $1-c/\mu_2+2c\leq 1$.

Proof. Directly,

$$\frac{\mu_2-c}{\mu_2}+2c=1-\frac{c}{\mu_2}+2c\leq 1-2c+2c=1,$$

since $\mu_2\leq\frac{1}{2}$. ∎

Claim B.5. $3c-2(\mu_1-c)\mu_2c/(\mu_2-c)\leq 2c$.

Proof. This holds if and only if

$$1\leq\frac{2(\mu_1-c)\mu_2}{\mu_2-c} \quad\Longleftrightarrow\quad \frac{\mu_2-c}{\mu_2}\leq 2(\mu_1-c),$$

which holds by assumption. ∎

Claim B.6. $F_{\frac{\mu_2-c}{\mu_2}}(x)$ *does not have support above* 1, *i.e.*, $1-c/\mu_2+3c-2(\mu_1-c)\mu_2c/(\mu_2-c)\leq 1$.

Proof. Directly,

$$\frac{\mu_2-c}{\mu_2}+3c-\frac{2(\mu_1-c)\mu_2}{\mu_2-c}c\leq\frac{\mu_2-c}{\mu_2}+2c\leq 1,$$

where the first inequality follows from Claim B.5, and the second inequality from Claim B.4. ∎

Claim B.7.

$$\frac{\mu_2-c}{\mu_2}+3c-\frac{2(\mu_1-c)\mu_2}{\mu_2-c}c>\frac{\mu_2-c}{\mu_2}\geq 2(\mu_1-c)-\frac{\mu_2-c}{\mu_2}.$$

Proof. The right hand inequality holds since $1-\mu_2/c\geq\mu_1-c$. Now the left-hand inequality:

$$\frac{\mu_2-c}{\mu_2}+3c-\frac{2(\mu_1-c)\mu_2}{\mu_2-c}c\geq\frac{\mu_2-c}{\mu_2}+\frac{3(\mu_1-c)\mu_2}{\mu_2-c}c-\frac{2(\mu_1-c)\mu_2}{\mu_2-c}c>\frac{\mu_2-c}{\mu_2},$$

since $1-c/\mu_2\geq\mu_1-c$. ∎

Note that for the special sub-case where $2(\mu_1 - c) - (\mu_2 - c) \geq 1 - c/\mu_2 > \mu_1 - c$, the distribution over effective values can also be generated by a pure strategy distribution over values F^*, where

$$F^*(x) = \begin{cases} 2\frac{\mu_2}{\mu_2 - c}\left(1 - \frac{(\mu_1 - c)\mu_2}{\mu_2 - c}\right)x & \text{if} \quad x \in \left[0, \frac{\mu_2 - c}{\mu_2}\right] \\ 2\left(1 - \frac{(\mu_1 - c)\mu_2}{\mu_2 - c}\right) & \text{if} \quad x \in \left[\frac{\mu_2 - c}{\mu_2}, \frac{\mu_2 - c}{\mu_2} + \frac{c(\mu_2 - c)}{2(\mu_1 - c)\mu_2 - (\mu_2 - c)}\right) \\ 1 & \text{if} \quad x \in \left[\frac{\mu_2 - c}{\mu_2} + \frac{c(\mu_2 - c)}{2(\mu_1 - c)\mu_2 - (\mu_2 - c)}, 1\right] \end{cases}.$$

It suffices to check

$$\frac{\mu_2 - c}{\mu_2} + \frac{c(\mu_2 - c)}{2(\mu_1 - c)\mu_2 - (\mu_2 - c)} \leq 1 \quad \Longleftrightarrow \quad 2(\mu_1 - c) - (\mu_2 - c) \geq 1 - \frac{c}{\mu_2}.$$

Firm 2's random (reservation) value U_2, is distributed according to distribution G_2:

$$G_2(u) \equiv \mathbb{P}(U_2 \leq u) = \frac{2\mu_2}{\mu_2 - c}u - 1, \quad \text{on} \quad \left[\frac{\mu_2 - c}{2\mu_2}, \frac{\mu_2 - c}{\mu_2}\right],$$

where for each $u \in [1/2 - c/(2\mu_2), 1 - c/\mu_2]$, F_u is given by the *ternary* distribution

$$F_u = \begin{Bmatrix} 0 & b(u) & a(u) \\ 1 - 2\mu_2 & \mu_2 & \mu_2 \end{Bmatrix}, \quad \text{where} \quad a(u) \equiv u + \frac{c}{\mu_2} \quad \text{and} \quad b(u) \equiv \frac{\mu_2 - c}{\mu_2} - u.$$

∎

Lemma B.8. *If $\mu_2 \geq 1/2$ and $1 - \mu_2/c \geq \mu_1 - c$, there is an equilibrium in which firm 1 chooses the binary distribution over effective values with support $\{0, 1 - c/\mu_2\}$, and firm 2 chooses the distribution over effective values $H_2(w)$, where*

$$H_2(w) = \begin{cases} \frac{(1-\mu_2)^2}{\mu_2}\left(\frac{w}{\mu_2 - c}\right) & \textit{if} \quad w \in [0, \mu_2 - c) \\ \frac{\mu_2}{\mu_2 - c}w & \textit{if} \quad w \in \left[\mu_2 - c, 1 - \frac{c}{\mu_2}\right] \\ 1 & \textit{if} \quad w \geq 1 - \frac{c}{\mu_2} \end{cases}.$$

Proof. Firm 1 induces its effective-value distribution by choosing the binary distribution over posteriors with support $\{0, \mu_1(\mu_2 - c)/(\mu_2(\mu_1 - c))\}$, and firm 2 induces its effective-value distribution by choosing distribution G_2 over reservation values:

$$G_2(u) \equiv \mathbb{P}(U_2 \leq u) = \frac{u}{\mu_2 - c} - \frac{1 - \mu_2}{\mu_2} \quad \text{on} \quad \left[\mu_2 - c, \frac{\mu_2 - c}{\mu_2}\right],$$

where for each $u \in [\mu_2 - c, 1 - c/\mu_2]$, F_u is binary with support $\{(\mu_2 - c - \mu_2 u)/(1 - \mu_2), u + c/\mu_2\}$.

∎

B.2 Proof of Proposition 6.3

The development of this result mirrors that for the no outside option case and follows from a sequence of lemmata.

Lemma B.9. *Suppose $n \geq 3$ and the consumer's outside option is relevant, i.e., $u_0 > 0$. Denote by H a symmetric-equilibrium distribution of effective values chosen by each firm, let $\alpha \in [0,1]$ be the probability that a firm offers full information, and let $\hat{U} \equiv \sup\left(supp\,(H)/\left\{\bar{U}\right\}\right)$. Then the payoff function in effective values facing each individual firm must have a linear structure that is either semi-linear:*

$$\Pi(w;H) = \begin{cases} 0 & \text{if } w = [0, u_0) \\ (\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U}-u_0} \times (w - u_0) & \text{if } w \in \left[u_0, \hat{U}\right] \\ (1-\alpha\mu)^{n-1} & \text{if } w \in (\hat{U}, \bar{U}) \\ \frac{1-(1-\alpha\mu)^n}{n\alpha\mu} & \text{if } w = \bar{U} \end{cases}, \tag{19}$$

with $(\alpha(1-\mu))^{n-1}/u_0 > \left[(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}\right] / \left(\hat{U} - u_0\right)$; or fully-linear:

$$\Pi(w;H) = \begin{cases} 0 & \text{if } w = [0, u_0) \\ (1-\alpha\mu)^{n-1} \times \frac{w}{\hat{U}} & \text{if } w \in \left[u_0, \hat{U}\right] \\ (1-\alpha\mu)^{n-1} & \text{if } w = \left(\hat{U}, \bar{U}\right) \\ 1 & \text{if } w = \bar{U} \end{cases}. \tag{20}$$

Proof. The linearity of the equilibrium payoff function over $\left(u_0, \hat{U}\right)$ can be shown by an argument similar to that of Lemma 4.2 after trivial adaptation, and is thus omitted. ∎

The symmetric equilibrium distribution of effective values can thus be characterized by parameters α and $\hat{U}$, as well as whether it induces a payoff function with a semi-linear form (19), or a fully-linear form (20). The following lemma shows that these parameters are uniquely pinned down via the dependence of the equilibrium payoff on the atom assigned to effective value 0.

Lemma B.10. *Suppose $n \geq 3$, and let μ^{FD} be the unique solution to equation $1-(1-\mu)^n = n(1-\mu)^{n-1}$. The symmetric equilibrium distribution of effective values is unique, and its form depends on the average quality μ and the outside option u_0 as follows.*

(i) If $\mu \geq \mu^{FD}$, the equilibrium has full disclosure for all $u_0 > 0$.

(ii) For each $\mu \in \left(1/n, \mu^{FD}\right)$, there are cutoffs u_0^L and u_0^{FD} such that the equilibrium payoff function necessarily takes the semi-linear form if $u_0 < u_0^L$, takes the fully-linear form if $u_0 \in [u_0^L, u_0^{FD})$, and has full disclosure if $u_0 \geq u_0^{FD}$.

(iii) For each $\mu \leq 1/n$, there is a cutoff u_0^{FD} such that the equilibrium payoff function necessarily takes the fully-linear form if $u_0 < u_0^{FD}$ and has full disclosure if $u_0 \geq u_0^{FD}$.

Proof of Lemma B.10. Consider first the case of full disclosure in equilibrium. If all other firms are fully revealing, the expected payoff of a firm by following suit is $\left(1 - (1-\mu)^n\right)/n$. The optimal deviation is either a distribution with support $\{\underline{U}\}$ (if $u_0 \leq \underline{U}$) or one with support $\{0, u_0\}$ (if $u_0 > \underline{U}$), with respective payoffs $(1-\mu)^{n-1}$ and $(1-\mu) \times \underline{U}/u_0$. Therefore, full disclosure can arise in equilibrium if and only if

$$\frac{1-(1-\mu)^n}{n} \geq (1-\mu)^{n-1} \times \min\left\{1, \frac{\underline{U}}{u_0}\right\}. \tag{21}$$

Recall that μ^{FD} is the unique solution to equation $1-(1-\mu)^n = n(1-\mu)^{n-1}$. Inequality (21) holds whenever $\mu \geq \mu^{FD}$ regardless of u_0. When $\mu < \mu^{FD}$, inequality (21) holds if and only if u_0 is sufficiently large; specifically:

$$u_0 \geq \frac{n(1-\mu)^{n-1}}{1-(1-\mu)^n} \times (\mu - c) \equiv u_0^{FD}(\mu).$$

Note that u_0^{FD} is hump-shaped with $u_0^{FD}(c) = u_0^{FD}(1) = 0$. Therefore, for each $u_0 \in \left(0, \bar{U}\right)$, full disclosure can be sustained as an equilibrium either if μ is sufficiently large or if μ is sufficiently small.

We now move onto partial disclosure equilibrium. Denote by v the equilibrium payoff of an individual firm, and by $\beta \in (0, 1-\mu)$ the atom at 0 that an individual firm assigns in its effective-value distribution. The two variables are related by

$$v = \frac{1-\beta^n}{n}. \tag{22}$$

Suppose the equilibrium takes the semi-linear form. In this case, reservation value $\bar{U}$ is on the support and must deliver the equilibrium payoff v. Moreover, the atom β at 0 is due only to reservation value $\bar{U}$, and hence is equal to $\alpha(1-\mu)$. These two facts imply

$$v = (1-\mu)\,\frac{1-\left(1-\frac{\mu}{1-\mu}\beta\right)^n}{n\beta}. \tag{23}$$

Equating (22) and (23) give an equation in β, which has a unique solution in the interval $(0, 1-\mu)$ if and only if $\mu > n^{-1}$. To see this, note that the RHS of (22) is decreasing and concave in β and equal to $1/n$ at $\beta = 0$, whereas the RHS of (23) is decreasing and convex in β and equal to μ at $\beta = 0$. Moreover, it is straightforward to verify that the RHS of the two equations coincide when $\beta = 1-\mu$.

Suppose $\mu > n^{-1}$ and denote the unique solution (in the interval $\beta \in (0, 1-\mu)$) to the system of equations (22) and (23) above by $\left(\hat{v}, \hat{\beta}\right)$. Suppose further that $u_0 \leq \underline{U}$. As $\Pi_{\underline{U}}(w) = \hat{\Pi}_{\underline{U}}(w)$ for all $w \in [u_0, \underline{U}]$ (otherwise, this interval of effective values would not be on the support of the equilibrium distribution), it is necessary that

$$\frac{\hat{\beta}^{n-1}}{u_0} \geq \frac{\hat{v}}{\underline{U}} \Longleftrightarrow u_0 \leq \frac{\hat{\beta}^{n-1}}{\hat{v}} \underline{U} \equiv u_0^L(\mu).$$

It is noteworthy that $u_0^L(\mu)$ equals 0 at $\mu = n^{-1}$, equals $\mu^{FD} - c$ at $\mu = \mu^{FD}$, and is increasing in μ. Moreover, it can be shown that $\hat{v} \geq \hat{\beta}^{n-1}$, so that $u_0^L(\mu) \leq \underline{U}$. In sum, the equilibrium can take the semi-linear form only if $\mu > n^{-1}$ and $u_0 \leq u_0^L(\mu)$. In this case, $\alpha = \hat{\alpha} \equiv \hat{\beta}/(1-\mu)$ and

$$\hat{U} - u_0 = \frac{\left(1 - \frac{\mu\hat{\beta}}{1-\mu}\right)^{n-1} - \hat{\beta}^{n-1}}{\hat{v} - \hat{\beta}^{n-1}} (\underline{U} - u_0). \tag{24}$$

The requirement $u_0 \leq u_0^L(\mu)$ ensures that $\frac{(\alpha(1-\mu))^{n-1}}{u_0} > \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}$ holds. We wish to establish that $\hat{U} \leq \bar{U}$. To this end, it suffices to focus on the case $u_0 = 0$, as $\hat{U}$ stated above is decreasing in u_0. As $\hat{v}$ and $\hat{\beta}$ are obtained by solving the system (22) and (23), the inequality $\hat{U} \leq \bar{U}$ can be stated as

$$\frac{1 - (1 - \mu\hat{\alpha})^n}{n} \geq \mu\hat{\alpha}(1 - \mu\hat{\alpha})^{n-1} + (\hat{\alpha}(1-\mu))^n, \tag{25}$$

where $\hat{\alpha}$ is implicitly given by $\hat{\alpha}(\hat{\alpha}(1-\mu))^n + (1-\hat{\alpha}) = (1-\mu\hat{\alpha})^n$. It follows from a change of variable and straightforward algebra that (25) can be rewritten as

$$T(x) \equiv \frac{\left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1} \left(\left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1} - 1 + x\right)^n}{x^n + \left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1} - 1} \leq 1, \tag{26}$$

where $x = 1 - \mu\hat{\alpha}$.[28] We show that (26) holds for all $x \in \left[0, \mu^{FD}\right]$ and $n \geq 3$. It follows from direct substitution that $T(0) = 0$ and $T\left(\mu^{FD}\right) = 1$. It remains to show that $T(x)$ is increasing. By direct

[28]Using the definition of $\hat{\alpha}$, the inequality $\hat{U} \leq \bar{U}$ can be written as $\hat{\alpha} \leq \left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1}$. Moreover, the

computation, $T'(x)$ has the same sign as $x^3 A(x) + x^{n+3} B(x)$, where

$$\begin{aligned} A(x) &= (1-x^n)(n-1) - x^{n-2}\left(n^2-n+1\right)(1-x) + x^{n-1}(1-x) \\ B(x) &= 1 - x^{n-3}(2x-1)(n+x-nx) \end{aligned}$$

We show that both $A(x)$ and $B(x)$ are nonnegative over $x \in \left[0, \mu^{FD}\right]$. First,

$$A'(x) = n^2 x^{n-3}(1-x)\left(x - \frac{n^3-3n^2+3n-2}{n^2}\right).$$

As $\left(n^3-3n^2+3n-2\right)/n^2$ exceeds μ^{FD} for all $n \geq 3$, $A(x)$ is decreasing. Moreover, $A(0) = n-1 > 0$ and $A(1) = 0$. Second,

$$B'(x) = -2(n-1)^2 x^{n-4}(1-x)\left(x - \frac{n(n-3)}{2(n-1)^2}\right)$$

Therefore, B is either increasing or inverted U-shaped. Moreover, $B(0) = 1$ and $B(1) = 0$.

Finally, consider equilibria that take the fully-linear form. With a full-linear payoff function, the equilibrium payoff v is equal to $\hat{\Pi}(\underline{U})$, implying that

$$v = \frac{\beta^{n-1}}{u_0}\underline{U}. \tag{27}$$

If the full information equilibrium exists, i.e., (21) holds, the solution to the system of equations (22) and (27) would have $\beta > 1-\mu$ and $v < (1-(1-\mu)^n)/n$, eliminating this class of equilibria. Therefore, for the rest of this proof, suppose (21) does not hold. It is clear that the system (22) and (27) has a unique solution–denote it by $\left(\hat{v}, \hat{\beta}\right)$. Define the probability of full information, $\hat{\alpha}$, as follows. If $\hat{v} \geq \mu$, set $\hat{\alpha} = 0$; otherwise, set $\hat{\alpha}$ to be the unique solution to $\hat{v} = \frac{1-(1-\alpha\mu)^n}{n\alpha}$.[29] Moreover, full-linearity dictates that $\hat{U} = (1-\hat{\alpha}\mu)^{n-1} \times \underline{U}/\hat{v}$. Evidently,

$$\hat{U} = (1-\alpha\mu)^{n-1} \times \frac{\underline{U}}{\hat{v}} = \frac{(1-\alpha\mu)^{n-1}}{\frac{1-(1-\alpha\mu)^n}{n\alpha}} \times \underline{U} = \frac{n\alpha(1-\alpha\mu)^{n-1}}{(1-(1-\alpha\mu)^n)} \times \underline{U} \leq \frac{1}{\mu} \times \underline{U} = \bar{U},$$

where the inequality follows from the fact that $n\alpha(1-\alpha\mu)^{n-1}/(1-(1-\alpha\mu)^n)$ is a decreasing function in n and is equal to μ^{-1} at $\alpha = 0$.

---

implicit definition of $\hat{\alpha}$ can transformed into $x^n + \hat{\alpha} - 1 - \hat{\alpha}(\hat{\alpha}+x-1)^n = 0$, yielding an inverse relation between $\hat{\alpha}$ and x. Moreover, as the LHS of the last equation is increasing in $\hat{\alpha}$, the inequality stated holds by substituting $\hat{\alpha} = \left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1}$.

[29] Note that failure of (21) ensures that $\hat{\alpha} < 1$.

We finish by verifying that a fully-linear payoff function with $\alpha = \hat{\alpha}$ and $\hat{U}$ chosen above satisfies the necessary conditions for an equilibrium whenever $\mu \leq n^{-1}$ or $u_0 \geq u_0^L(\mu)$. To this end, it suffices to check that $\hat{\beta} \geq \hat{\alpha} \times (1-\mu)$. The case of $\hat{\alpha} = 0$ ($\hat{v} \geq \mu$) is immediate, so consider $\hat{\alpha} > 0$. As the RHS of (22) is decreasing and concave in β, whereas the RHS of (23) is decreasing and convex in β, either $\mu \leq n^{-1}$ or $u_0 \geq u_0^L(\mu)$ ensures that

$$\hat{v} \geq (1-\mu)\frac{1-\left(1-\frac{\mu}{1-\mu}\hat{\beta}\right)^n}{n\hat{\beta}}. \tag{28}$$

The fact that $\frac{1-(1-\alpha\mu)^n}{n\alpha}$ is decreasing in α, together with the definition $\hat{v} = \frac{1-(1-\hat{\alpha}\mu)^n}{n\hat{\alpha}}$, implies that $\hat{\beta}/(1-\mu) \geq \hat{\alpha}$.

The analysis above covers all parameter configurations for any $n \geq 3$, $\mu \in (0,1)$ and $u_0 \in \left(0, \bar{U}\right)$. ■

Similar to the no outside option case, the effective-value distributions characterized in Lemmata B.9 and B.10 can be generated by mixed strategies that involve randomization of binary distributions over posteriors only.

Lemma B.11. *The effective-value distribution H implied by the payoff function in Lemma 4.2, with α and $\hat{U}$ given by Lemma B.10, is inducible. Moreover, it can be generated by a mixed strategy $\left(G(\cdot), \{F_U(\cdot)\}_{U \in supp(G)}\right)$ in which F_U is binary for each $U \in supp(G)$.*

Proof of Lemma B.11. Consider first the case $\mu \leq n^{-1}$ and $u_0 \in [u_0^L(\mu), u_0^{FD}(\mu))$, so that the equilibrium payoff function is fully linear. The effective-value distribution H implied by (20) has an atom α at $\bar{U}$, an atom $(1-\alpha\mu)\left(\frac{u_0}{\hat{U}}\right)^{\frac{1}{n-1}}$ at 0, and a density

$$h(w) = \begin{cases} 0 & \text{if } w < u_0 \text{ and } w = (\hat{U}, \bar{U}] \\ \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} w^{-\frac{n-2}{n-1}} & \text{if } w \in \left[u_0, \hat{U}\right] \end{cases}.$$

Below, we construct a mixed strategy that generates this effective value distribution. To this end, define a mapping $b : \left[\underline{U}, \hat{U}\right] \longrightarrow [0, \underline{U}]$ by

$$K(b(U)) = K(U) \text{ for } U \leq \tilde{U}, \text{ and } b(U) = 0 \text{ for } U \geq \tilde{U},$$

where $K : \left[0, \bar{U}\right] \longrightarrow \mathbb{R}$ is defined as $K(w) \equiv (n\underline{U} - w)\, w^{\frac{1}{n-1}}$, $\tilde{U} > u_0$ is defined implicitly by $K\left(\tilde{U}\right) = K(u_0)$, and parameters α and $\hat{U}$ are as given in Lemma B.10. For each $U \in \left[\underline{U}, \hat{U}\right]$, let F_U be the binary distribution with support $\{b(U), U\}$ and mean $\underline{U}$ and let $F_{\bar{U}}$ be the binary distribution with support $\left\{0, \bar{U}\right\}$ and mean $\underline{U}$. Moreover, let G be a reservation-value distribution that has an atom $\alpha \in [0,1]$ at $\bar{U}$ and a density as follows:

$$g(U) \equiv \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{U-b(U)}{\underline{U}-b(U)} U^{-\frac{n-2}{n-1}}, \text{ for } U \in \left[\underline{U}, \hat{U}\right].$$

Below, we show that the mixed strategy $\left(G, \{F_U(\cdot)\}_{U \in \left[\underline{U}, \hat{U}\right] \cup \left\{\bar{U}\right\}}\right)$ generates the effective-value distribution H defined above.

We need to establish that the effective-value distribution F_U is inducible for each $U \in \left[\underline{U}, \hat{U}\right] \cup \left\{\bar{U}\right\}$. First, the mapping b is well-defined: a direct computation reveals that $K(w)$ is strictly concave with its peak at $\underline{U}$. Moreover, we can show that $b(U) \leq \bar{a}(U) \equiv \frac{\mu-c-\mu U}{1-c-U}$ (as defined in Lemma 3.6). For this purpose, it is without loss to suppose $u_0 = 0$, as $b(U)$ defined above is weakly decreasing in u_0. Because $\bar{a}(\underline{U}) = b(\underline{U})$, $b\left(\hat{U}\right) = 0 = \bar{a}\left(\bar{U}\right) \leq \bar{a}\left(\hat{U}\right)$, and $\bar{a}(U)$ is decreasing and strictly concave, it suffices to show that $b(U)$ is convex. To this end, we adopt a change of variable: let $v = U - \underline{U}$, and $d(v) = \underline{U} - b(\underline{U} + v)$. The implicit definition of b implies $K(\underline{U} + v) = K(\underline{U} - d(v))$, or equivalently,

$$(\underline{U} + v)^{\frac{1}{n-1}} ((n-1)\underline{U} - v) = (\underline{U} - d(v))^{\frac{1}{n-1}} ((n-1)\underline{U} + d(v)).$$

The rest of the argument coincides with that in Lemma 4.4 (after replacing M with $\underline{U}$).

We now check that the mixed strategy generates an effective-value distribution coinciding with H stated above. For $w \in \left[\underline{U}, \tilde{U}\right]$, the density implied by the mixed strategy is

$$g(w) \times \frac{\underline{U} - b(w)}{w - b(w)} = \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{w-b(w)}{\underline{U}-b(w)} w^{-\frac{n-2}{n-1}} \times \frac{\underline{U}-b(w)}{w-b(w)} = h(w).$$

For $w \in \left[\tilde{U}, \hat{U}\right]$, the density implied by the mixed strategy is

$$g(w) \times \frac{\underline{U}}{w} = \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{w}{\underline{U}} w^{-\frac{n-2}{n-1}} \times \frac{\underline{U}}{w} = h(w).$$

Define $q : [0, \underline{U}] \longrightarrow \left[\underline{U}, \hat{U}\right]$ as the inverse mapping of b. For $w \in [u_0, \underline{U}]$, the density implied by the mixed strategy is

$$
\begin{aligned}
& -q'(w) \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \quad & -\frac{K'(w)}{K'(q(w))} \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \quad & -\frac{\frac{n}{n-1}\left(\underline{U} - w\right) w^{\frac{1}{n-1}-1}}{\frac{n}{n-1}\left(\underline{U} - q(w)\right) q(w)^{\frac{1}{n-1}-1}} \times \frac{q(w) - \underline{U}}{q(w) - w} \times \frac{1 - \alpha\mu}{(n-1)\,\hat{U}^{\frac{1}{n-1}}} \frac{q(w) - w}{\underline{U} - w} q(w)^{-\frac{n-2}{n-1}} \\
= \quad & \frac{w^{\frac{1}{n-1}-1}}{q(w)^{\frac{1}{n-1}-1}} \times \frac{\underline{U} - w}{q(w) - w} \times \frac{1 - \alpha\mu}{(n-1)\,\hat{U}^{\frac{1}{n-1}}} \frac{q(w) - w}{\underline{U} - w} q(w)^{-\frac{n-2}{n-1}} \\
= \quad & \frac{1 - \alpha\mu}{(n-1)\,\hat{U}^{\frac{1}{n-1}}} \times w^{-\frac{n-2}{n-1}} = h(w).
\end{aligned}
$$

The atom at 0 is given by

$$
\begin{aligned}
\alpha(1-\mu) + \int_{\tilde{U}}^{\hat{U}} \left(1 - \frac{\underline{U}}{U}\right) dG(U) &= \alpha(1-\mu) + \int_{\tilde{U}}^{\hat{U}} \left(1 - \frac{\underline{U}}{w}\right) \times \left(\frac{1 - \alpha\mu}{(n-1)\,\hat{U}^{\frac{1}{n-1}}} \frac{w}{\underline{U}} w^{-\frac{n-2}{n-1}}\right) dw \\
&= \alpha(1-\mu) + \frac{1 - \alpha\mu}{n\underline{U}\hat{U}^{\frac{1}{n-1}}} \left(-\left(n\underline{U} - \hat{U}\right)\hat{U}^{\frac{1}{n-1}} + \left(n\underline{U} - \tilde{U}\right)\tilde{U}^{\frac{1}{n-1}}\right) \\
&= \alpha(1-\mu) + \frac{1 - \alpha\mu}{n\underline{U}\hat{U}^{\frac{1}{n-1}}} \left(-\left(n\underline{U} - \hat{U}\right)\hat{U}^{\frac{1}{n-1}} + \left(n\underline{U} - u_0\right) u_0^{\frac{1}{n-1}}\right) \\
&= \alpha(1-\mu) + (1 - \alpha\mu)\left(-\left(1 - \frac{(1-\alpha\mu)^{n-1}}{nv}\right) + \left(1 - \frac{\beta^{n-1}}{nv}\right)\frac{\beta}{1 - \alpha\mu}\right) \\
&= \beta + \frac{(1-\alpha\mu)^n - \beta^n - (1-\alpha)\, nv}{nv} \\
&= \beta.
\end{aligned}
$$

where the first equality uses the definition of g, the third equality uses the definition of $\tilde{U}$, the fourth equality uses the linearity of the payoff function: $v/\underline{U} = \beta^{n-1}/u_0 = (1-\alpha\mu)^{n-1}/\hat{U}$, and the last equality uses the fact that $v = \frac{1-\beta^n}{n}$ and $v = \frac{1-(1-\alpha\mu)^n}{n\alpha}$ (in the case $\alpha > 0$).

Consider next the case $\mu > n^{-1}$ and $u_0 \in [0, u_0^L(\mu))$, so that the equilibrium payoff function is semi-linear. The effective-value distribution H implied by (19) has an atom α at $\bar{U}$, an atom $\alpha(1-\mu)$ at 0, and a density

$$
h(w) = \begin{cases} 0 & \text{if } w < u_0 \text{ and } w = (\hat{U}, \bar{U}] \\ \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)} \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}} & \text{if } w \in \left[u_0, \hat{U}\right] \end{cases}.
$$

Define a mapping $b : \left[\underline{U}, \hat{U}\right] \longrightarrow [0, \underline{U}]$ by $K(b(U)) = K(U)$, where $K : \left[0, \bar{U}\right] \longrightarrow \mathbb{R}$ is given by

$$
\begin{aligned}
K(w) \quad \equiv \quad & \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0} (w - u_0) \right)^{\frac{1}{n-1}} \\
& \times \left(n\underline{U} - (n-1) u_0 + (n-1) \left(\frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0} \right)^{-1} (\alpha(1-\mu))^{n-1} - w \right),
\end{aligned}
$$

and parameters α and $\hat{U}$ are as given in Lemma B.10. For each $U \in \left[\underline{U}, \hat{U}\right]$, let F_U be a binary distribution with support $\{b(U), U\}$ and mean $\underline{U}$, and let $F_{\bar{U}}$ be the binary distribution with support $\left\{0, \bar{U}\right\}$ and mean $\underline{U}$. Moreover, let G be a reservation-value distribution, which has an atom $\alpha \in [0,1]$ at $\bar{U}$, and a density as follows:

$$
\begin{aligned}
g(U) \quad \equiv \quad & \tfrac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)} \left((\alpha(1-\mu))^{n-1} + \tfrac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0} (U - u_0) \right)^{-\frac{n-2}{n-1}} \\
& \times \tfrac{U - b(U)}{\underline{U} - b(U)}, \qquad \text{for } U \in \left[\underline{U}, \hat{U}\right].
\end{aligned}
$$

We need to establish that effective-value distribution F_U is inducible for each $U \in \left[\underline{U}, \hat{U}\right] \cup \left\{\bar{U}\right\}$. First, the mapping b is well-defined: a direct computation reveals that $K(w)$ is strictly concave with its peak at $\underline{U}$ and that $K\left(\hat{U}\right) = K(u_0)$. Moreover, we can show that $b(U) \leq \bar{a}(U) \equiv \frac{\mu - c - \mu U}{1 - c - U}$ (as defined in Lemma 3.6). To this end, note that because $\bar{a}(\underline{U}) = b(\underline{U})$, $b\left(\hat{U}\right) = u_0 \leq \bar{a}\left(\hat{U}\right)$, [30] and $\bar{a}(U)$ is decreasing and strictly concave, it suffices to show that $b(U)$ is convex. To this end, we adopt a change of variable: let $v = U - \underline{U}$, and $d(v) = \underline{U} - b(\underline{U} + v)$. The implicit definition of b implies $K(\underline{U} + v) = K(\underline{U} - d(v))$, or equivalently,

$$
(L + v)^{\frac{1}{n-1}} ((n-1)L - v) = (L - d(v))^{\frac{1}{n-1}} ((n-1)L + d(v)),
$$

where $L \equiv (\underline{U} - u_0) + \frac{\left(\hat{U} - u_0\right)(\alpha(1-\mu))^{n-1}}{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}$. The rest of the argument coincides with that in Lemma 4.4 (after replacing M with L).

Let us now check that the mixed strategy generates an effective-value distribution coinciding

[30] The reason is as follows. The derivation in Lemma B.10 reveals that $\hat{U} \leq \bar{U}$ in this parameter range, including the case $u_0 = 0$, so $b^{-1}(0) \leq \bar{U}$. Moreover, we have established in the case of the fully-linear equilibrium above that $b\left(\hat{U}\right) \leq \bar{a}\left(\hat{U}\right)$ holds at $u_0 = u_0^L(\mu)$, so $b^{-1}\left(u_0^L(\mu)\right) \leq \bar{a}^{-1}\left(u_0^L(\mu)\right)$. Furthermore, for all $u_0 \in [0, u_0^L(\mu))$, $\hat{U}$ varies linearly with u_0. Together with the fact that $\bar{a}$ is strictly concave, we have $b^{-1}(u_0) < \bar{a}^{-1}(u_0)$ for all $u_0 \in [0, u_0^L(\mu))$.

with H stated above. For $w \in \left[\underline{U}, \hat{U}\right]$, the density implied by the mixed strategy is

$$\begin{aligned}
& g(w) \times \frac{\underline{U} - b(w)}{w - b(w)} \\
= \quad & \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)} \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}} \\
& \times \left(\frac{w - b(w)}{\underline{U} - b(w)}\right)\left(\frac{\underline{U} - b(w)}{w - b(w)}\right) = h(w).
\end{aligned}$$

Let q be the inverse mapping of b. For $w \in [u_0, \underline{U}]$, the density implied by the mixed strategy is

$$\begin{aligned}
& -q'(w) \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \quad & -\frac{K'(w)}{K'(q(w))} \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \quad & -\frac{(\underline{U} - w)\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}}}{(\underline{U} - q(w))\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(q(w) - u_0)\right)^{-\frac{n-2}{n-1}}} \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= \quad & \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)} \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}} \\
= \quad & h(w).
\end{aligned}$$

■

B.3 Proof of Corollary 6.4

(i) Fix a $u_0 < \mu^{FD} - c$ and let μ^* be the unique solution to $u_0^L(\mu) = u_0$. We will show that the firm's equilibrium profit is increasing in μ when the equilibrium takes the fully linear form (i.e., $\mu < \mu^*$) and is decreasing in μ when the equilibrium takes the semi-linear form.

Consider first the case where the equilibrium payoff is fully linear. Recall from the proof of Lemma B.10 that the firm's equilibrium profit v is jointly determined by (22) and (27). As the RHS of (22) is decreasing in β and the RHS of (27) is increasing in β, and because an increase in μ shifts up the RHS of (27), the implied equilibrium payoff v is therefore increasing in μ.

Consider next the case where the equilibrium payoff is semi-linear. Recall from the proof of Lemma B.10 that the firm's equilibrium profit v and the atom β at the bottom is jointly determined by equating (22) and (23), i.e., $T(\beta,\mu) = 0$, where

$$T(\beta,\mu) \equiv (1-\mu)\frac{1-\left(1-\frac{\mu}{1-\mu}\beta\right)^n}{n\beta} - \frac{1-\beta^n}{n}.$$

It is straightforward to verify that T is strictly convex in β, is positive at $\beta = 0$ and equals 0 at $\beta = 1-\mu$. The root that is smaller than $1-\mu$ thus gives the equilibrium value of β. It is immediate that T is increasing in μ, and so is the equilibrium value of β.[31] As the equilibrium payoff is decreasing in β (recall (22)), it is also decreasing in μ.

(ii) It is immediate from the proof of Lemma B.10 that the search cost c has no impact on the firm's profit v if the equilibrium payoff function is semi-linear, or if the equilibrium involves full disclosure. In the case of a fully-linear equilibrium payoff, v is jointly determined by (22) and (27). It is immediate that (22) is independent of c whereas the RHS of (27) is decreasing in c. An increase in c thus lowers the equilibrium value of v.

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[31] Direct computation shows that T is increasing in μ if and only if $(1-\mu)\left(1-\left(1-\frac{\mu}{1-\mu}\beta\right)^n\right)/n\beta < \left(1-\frac{\mu}{1-\mu}\beta\right)^{n-1}$ holds for all $\beta \in (0, 1-\mu)$. Letting $\alpha = \beta/(1-\mu)$, the last inequality can be equivalently expressed as $(1-\alpha\mu)^n + n\alpha(1-\alpha\mu)^{n-1} - 1 > 0$ for all $\alpha \in (0,1)$. The last inequality holds because its LHS is inverted U-shaped in α, equal to 0 at $\alpha = 0$, and is positive at $\alpha = 1$ (as long as $\mu < \mu^{FD}$).

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