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Wealth Inequality and Endogenous Growth

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Keywords: wealth inequality, productivity slowdowns, aggregate demand, real interest rates, transition dynamics.

JEL Codes: E20, O40, G51, C61.

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1 Introduction

Advanced economies have been experiencing productivity slowdowns and rising inequality in recent decades. This paper investigates the relationship between these two trends by developing and analyzing an analytically tractable endogenous growth model with heterogeneous households.

Demand intensity, defined as the aggregate consumption-to-wealth (inclusive of income) ratio, connects rising inequality with productivity slowdowns. Intuitively, because rich households save more, rising wealth inequality yields a decrease in demand intensity. This change has a negative effect on the size of the market in which firms operate and reduces profit gains from improved production technology. As profit-maximizing firms have fewer incentives to spend on R&D, productivity growth slows. I emphasize this channel based on simple, tractable model equations and a quantitative investigation into the model dynamics.

R&D is the engine of productivity and economic growth, as in the seminal theories developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). On the supply side of the model, profit-maximizing firms make R&D decisions by comparing potential profits and costs. When aggregate consumption decreases, spending on R&D becomes less attractive because the market size and potential profits decrease. Thus, R&D intensity, defined as the ratio of R&D investments to the R&D stock following Benigno and Fornaro (2018), decreases and economic growth endogenously slows.

Households are subject to uninsurable idiosyncratic income risk, as in Bewley (undated), Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), and Kaplan, Moll and Violante (2018). In this framework of heterogeneous households with constant relative risk aversion (CRRA) preferences, consumption is concave in wealth inclusive of income (see Carroll and Kimball, 1996). Because rich households save more (see Dynan, Skinner and Zeldes, 2004), an increase in wealth (inclusive of income) inequality generates a persistent decrease in aggregate consumption relative to wealth. Then, in general equilibrium, the fall in demand intensity on the household side affects productivity growth by altering market size and R&D decisions.

I underscore this channel from rising wealth inequality through demand intensity and R&D intensity to productivity growth at the aggregate level using an analytically tractable characterization of the general equilibrium for the model economy. To embed the incomplete-market framework with heterogeneous households into a general equilibrium endogenous growth model, I employ the analytically tractable representation of individual consumption decisions and cross-sectional distributions of wealth inclusive of income from Lee (2021). This framework allows me to investigate the aggregate implications of distributional elements without oversimplifying household heterogeneity and sacrificing the tractability of the results.

For the quantitative analysis, I compute the transition dynamics from a low-inequality economy in the early 1980s to a high-inequality economy in the late 2010s. This general equilibrium transition path confirms the theoretical predictions. Given the realistic, gradual increase in wealth (inclusive of income) inequality, demand intensity and economic growth rates decrease, consistent with the empirical time series for advanced economies during the same period (Section 2). Furthermore, because rising wealth inequality is induced by an increase in idiosyncratic income risk in the model, households have a strong precautionary motive in the high-inequality economy. This increased precautionary motive in combination with growth slowdowns generates a secular downward trend in real interest rates, consistent with the data (see Laubach and Williams, 2003, 2016). Finally, as consumption demand decreases, savings increase relative to wealth. Thus, the capital-to-net national income ratio increases, as documented by Piketty and Zucman (2014) for rich countries.

I document that the welfare cost of rising wealth inequality is substantial. To maintain social welfare in the pretransition low-inequality equilibrium, individual consumption should increase by 19% in every period along the general equilibrium transition path towards a high-inequality economy. This cost originates from two sources: the direct effects on consumption inequality and average consumption and the general equilibrium effects on slow growth. Note that these variations are consistent with the empirical trends (see, e.g., Attanasio and Pistaferri, 2014; Attanasio, Hurst and Pistaferri, 2015; Fernald, 2015; Antolin-Diaz, Drechsel and Petrella, 2017). I find that the welfare cost is nearly equally split between these two factors. Thus, by looking through the lens of the heterogeneous household endogenous growth model, I conclude that distributional factors have substantial productivity and welfare implications.

The mechanism emphasized in this paper features a channel that links demand-side factors to firm R&D spending and productivity growth. This channel is consistent with empirical evidence based on firm-level data (see, e.g. Acemoglu and Linn, 2004; Bustos, 2011; Jaravel, 2019; Aghion et al., 2020). Ignaszak and Sedlácek (2021) develop an endogenous growth model with heterogeneous firms and highlight the market size effects on firm incentives to innovate. This paper complements the results in Ignaszak and Sedlácek (2021) and focuses on the growth implications of heterogeneous households. My framework emphasizes the role of rising inequality as a source of aggregate variations in demand and business investments in R&D. Relatedly, Comin and Gertler (2006), Barlevy (2007), Anzoategui et al. (2019), and Bianchi, Kung and Morales (2019) investigate the endogenous dynamics of R&D investment and their business cycle implications. This paper supplements their results by concentrating on secular, trend-level changes.

Productivity slowdowns are tightly related to low real interest rates and secular stagnation (see, e.g., Eichengreen, 2015; Gordon, 2015; Benigno and Fornaro, 2018; Eggertsson, Mehrotra and Robbins, 2019; Lunsford and West, 2019). I present wealth inequality as another relevant factor underlying these macroeconomic trends and complement other explanations for the slow growth and low real interest rates observed in the data. In related papers focusing on inequality, Straub (2018) and Mian, Straub and Sufi (2020, 2021) examine the effects of income inequality on real interest rates. This paper focuses on the inequality in wealth inclusive of income and its productivity implications as well as real interest rates.

I combine heterogeneous household models (e.g., Bewley, undated; Huggett, 1993; Aiyagari, 1994) with endogenous growth models. This framework enables me to connect demand-side (household) trends to slow-moving components on the supply side (firm). Using an AK-type production environment with heterogeneous households, Clemens and Heinemann (2015) conduct a related quantitative exercise. In contrast, my model builds on R&D-based productivity growth, as in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Additionally, I employ the analytically tractable model of heterogeneous households from Lee (2021). This paper utilizes the results in Lee (2021) to enhance the tractability of a general equilibrium model with endogenously determined productivity growth rates and wealth inequality, whereas Lee (2021) applies the framework to a neoclassical growth model. The important precursors of this tractable heterogeneous household model include Constantinides and Duffie (1996) and Heathcote, Storesletten and Violante (2014). My model augments their no-trade equilibrium with a nontrivial amount of asset trading among households and of the capital used in production.

The remainder of this paper is organized as follows. Section 2 describes the trend-level changes in inequality, demand intensity, R&D intensity, and economic growth in advanced economies since the 1980s. Section 3 develops a general equilibrium endogenous growth model with heterogeneous households and incomplete financial markets. Section 4 characterizes the general equilibrium of the model. I illustrate how demand intensity connects wealth inequality with productivity growth using analytical results. Section 5 presents the quantitative analysis. Based on the calibrated model, I investigate the comovement of productivity growth, demand intensity, real interest rates, and wealth inequality along the transition path from a low-inequality economy to a high-inequality economy. Furthermore, I investigate the welfare implications of rising wealth inequality. Section 6 concludes the paper. Proofs are relegated to Appendix A.

2 Suggestive empirical evidence

This section presents the empirical evidence on long-run changes in advanced economies since the 1980s. I show trends in macroeconomic time series for several rich countries to provide an international perspective on the results. I illustrate that trend-level comovement exists among the measures for wealth and income inequality, consumption demand relative to wealth, R&D investment relative to the R&D stock, and economic growth in these countries.

Following Piketty and Zucman (2014), my sample of advanced economies includes the G7 countries (Canada, France, Germany, Italy, Japan, the UK, and the US) and Australia. Panels (a) and (b) show the top 10%'s wealth and income shares, respectively. Clearly, wealth and income inequality have substantially increased since the 1980s in these countries, as noted by, e.g., Piketty and Saez (2003), Atkinson, Piketty and Saez (2011), Saez and Zucman (2016), and Garbinti, Goupille-Lebret and Piketty (2021). These changes in the cross-sectional allocation of economic resources are accompanied by slowdowns in GDP growth (panel (c)). Consistent with the results for the US and the Euro area in Fernald (2015), Antolin-Diaz, Drechsel and Petrella (2017), and Fernald and Inklaar (2020), these downward trends in GDP growth started prior to the global financial crisis in the late 2000s, implying the existence of causes other than the recent recessions. Panel (d) illustrates the concurrent significant decreases in the aggregate consumption-to-net private wealth ratio that represents demand intensity. Finally, I show R&D intensity in panel (e), which is a major driver of productivity growth in seminal endogenous growth models (e.g., Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) and in US data (Kung and Schmid, 2015). During the sample period, R&D intensity, defined as the ratio of business investments in R&D to the R&D stock as in Benigno and Fornaro (2018), gradually decreased by 3.4 p.p. on average among the eight countries.[1]

[1]To emphasize the trend-level dynamics, in Figure 1, I show 9-year moving averages for the real GDP per capita growth rates (panel (c)) and R&D intensity (panel (e)). Appendix B.1 contains data sources and

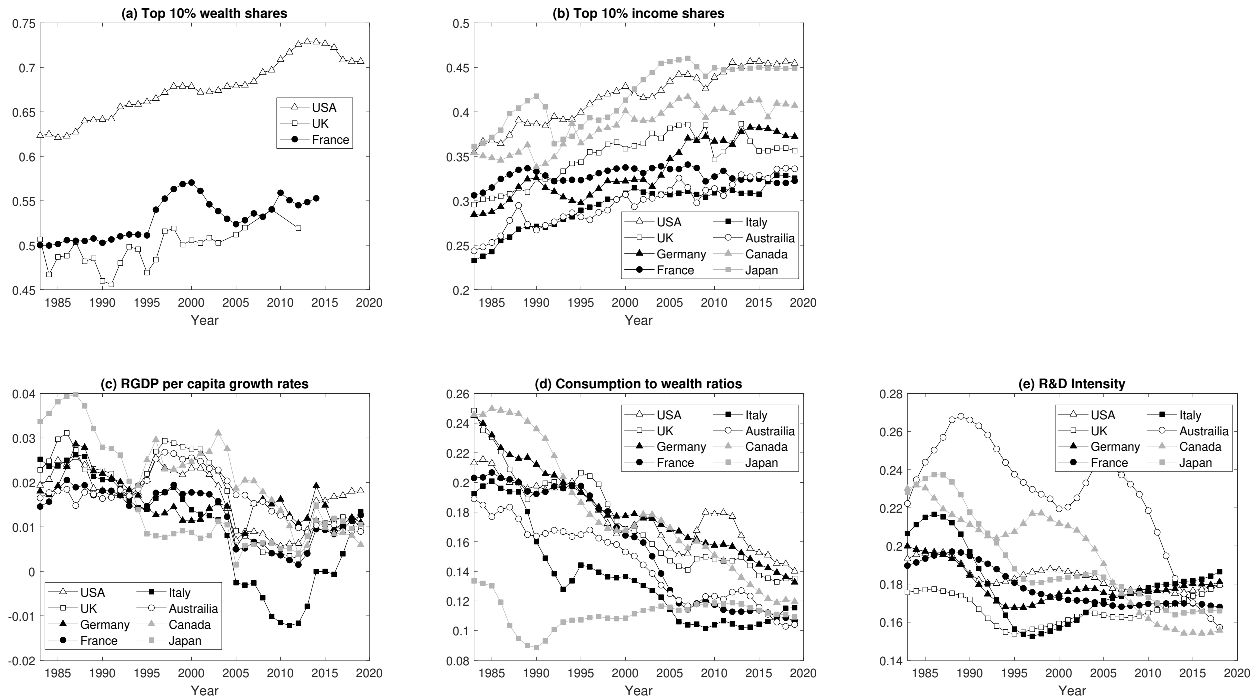

Figure 1: Inequality, consumption, R&D, and growth in rich countries

Notes: Panels (a) and (b) show the top 10%'s wealth and income shares, respectively. In panel (c), I show the 9-year moving averages for the growth rates in real GDP per capita to emphasize long-run dynamics. The consumption-to-wealth ratios in panel (d) are based on private consumption and private net wealth. Finally, R&D intensity in panel (e) is the ratio of business investments in R&D to the R&D stock. Similar to panel (c), panel (e) depicts 9-year moving averages for the original series. Following Piketty and Zucman (2014), my sample of advanced economies includes the G7 countries (Canada, France, Germany, Italy, Japan, the UK, and the US) and Australia. Appendix B.1 contains data sources and the details on the construction of these series.

This set of common stylized facts about advanced economies calls for a theoretical framework that can illustrate the relationships among these macroeconomic trends. In the remainder of this paper, I develop an endogenous growth model with heterogeneous households and emphasize a channel that links an increase in wealth and income inequality to GDP growth slowdowns through a decrease in demand intensity and a fall in R&D intensity.

3 Model

This section presents an endogenous growth model with heterogeneous households. By incorporating household heterogeneity into an endogenous growth framework, I build a general

the details on the construction of these series.

equilibrium model that relates cross-sectional inequality and aggregate economic growth. In the model, productivity growth is endogenously determined by firms' optimal hiring of R&D workers. This optimal decision depends on the market size and the potential profit from R&D. Note that when households are heterogeneous, market size (i.e., consumption demand) relies on the cross-sectional distribution of economic resources as well as aggregate quantities. Thus, in my model, wealth and income inequality may affect economic growth through their effects on consumption demand and the profitability of R&D.

To focus on this productivity implication of household heterogeneity, I assume that economic agents have perfect foresight regarding the aggregate variables. However, individual households are subject to uninsurable idiosyncratic income risk as in standard incomplete market models, such as those in Bewley (undated), Huggett (1993), and Aiyagari (1994) and death shocks as in Yaari (1965) and Blanchard (1985). The economy is closed and populated by a continuum of households and firms. When an agent dies, the government confiscates accidental bequests and redistributes them to newly born agents. Time is discrete.

3.1 Firms

There exists a continuum of intermediate goods $Y_{j,t}$, indexed by $j \in [0,1]$. The final good Y_t in period t is produced by combining $Y_{j,t}$ in a competitive market:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\omega-1}{\omega}} dj \right)^{\frac{\omega}{\omega-1}},$$

where ω is the elasticity of substitution. The price of the final good, P_t, is given by

$$P_t = \left(\int_0^1 P_{j,t}^{1-\omega} dj \right)^{\frac{1}{1-\omega}},$$

where $P_{j,t}$ is the price of intermediate good j. P_t is normalized to 1 in each period. The demand for intermediate good j follows from this structure:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\omega} Y_t. \tag{1}$$

Firm j produces $Y_{j,t}$ using the following production function:

$$Y_{j,t} = K_{j,t}^{1-\alpha} (A_{j,t} L_{Y,j,t})^{\alpha}, \tag{2}$$

where $A_{j,t}$ represents the productivity of firm j and $L_{Y,j,t}$ is the labor input into production. Capital $K_{j,t}$ is rented at a rate of $r_t + \delta$, where δ is the depreciation rate. By spending labor input $L_{A,j,t}$ on R&D, this firm can improve its production technology as follows:

$$A_{j,t+1} = (1 + \theta L_{A,j,t})A_{j,t}, \tag{3}$$

where θ measures the efficiency of R&D. Finally, firms pay a per-period administrative overhead labor cost, $W_t L_M$, where W_t is the real wage rate. Firm j's real profit in period t is given by

$$\Pi_{j,t} = \frac{P_{j,t}}{P_t} Y_{j,t} - W_t(L_{Y,j,t} + L_{A,j,t} + L_M) - (r_t + \delta)K_{j,t}. \tag{4}$$

Firms discount future profits using the real interest rate r and maximize the present discounted value of those future profits by choosing $\{P_{j,\tau}, K_{j,\tau}, L_{Y,j,\tau}, L_{A,j,\tau}, A_{j,\tau+1} : \tau \geq t\}$ given $A_{j,t}$. In equilibrium, profits are zero, preventing the entry of new firms.

In this model, the engine of growth is productivity-enhancing R&D conducted by incumbent firms. These firms spend on R&D in pursuit of additional profits originating from monopolistic competition and a better production technology that decreases the marginal cost of production.[2]

The productivity process (3) is nonstochastic. This nonstochastic specification is assumed to induce a uniform evolution of productivity across symmetric firms rather than a distribution of heterogeneous firms. This simple structure for the supply side enables me to preserve the analytical tractability of the equilibrium with endogenous growth, although incomplete markets and rich heterogeneity are introduced into the household side. Note further that at the aggregate level, the productivity growth rate g_t equals $\theta L_{A,t}$, as in the seminal R&D-based endogenous growth models developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) (see also Jones, 1995, Section II).

3.2 Households

The household side consists of a continuum of heterogeneous households, indexed by $i \in [0, 1]$. In an otherwise standard incomplete market model similar to those in Bewley (undated),

[2] Relatedly, in the data, incumbent firms contribute substantially to productivity growth (see Bartelsman and Doms, 2000, Section 3).

Huggett (1993), and Aiyagari (1994), I introduce a judiciously chosen process for idiosyncratic income risk that yields a simple but realistic optimal consumption function in wealth (inclusive of income) at the household level. This consumption function, combined with the tractable, realistic cross-sectional distributions of wealth, enables the analytical aggregation of individual decision and state variables in this model. This tractable incomplete market framework is adopted from Lee (2021). For exposition and completeness in this paper, I recapitulate several of the results from Lee (2021) although detailed discussions on the shape of the consumption function and the interpretation of the individual income process are kept to a minimum.

Preferences. Preferences are time-separable and feature CRRA:

$$U_{i,t} = \mathbb{E}_t \left[\sum_{\tau \geq t} \beta^{\tau - t} \frac{C_{i,\tau}^{1-\gamma} - 1}{1-\gamma} \right],$$

where C denotes consumption and γ is the relative risk aversion coefficient. As in Yaari (1965) and Blanchard (1985), agents may die with probability p_d in each period. The discount rate β is given by $\tilde{\beta}(1-p_d)$, where $\tilde{\beta}$ and $1-p_d$ reflect the subjective discount factor and the survival probability, respectively.

Budget constraints. Utility is maximized subject to the budget constraint:

$$C_{i,t} + K_{i,t+1} = (1+r_t)K_{i,t} + W_t \varepsilon_{i,t}, \tag{5}$$

where $K_{i,t}$ represents the assets owned by household i in period t, r_t is the risk-free rate, and W_t is the real wage rate per efficiency unit of labor. The endowment of labor with idiosyncratic productivity $\varepsilon_{i,t}$ is inelastically supplied. $M_{i,t}$ denotes the total stochastic income, $M_{i,t} = W_t \varepsilon_{i,t}$.

Let g_t be the growth rate of labor-augmenting productivity at the aggregate level ($g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$). The gross growth rate is denoted by $G_t = 1 + g_t$. I scale the variables by the growth rates and write them in small letters, e.g., $c_{i,t} \equiv \frac{C_{i,t}}{G_1 \times G_2 \times \ldots \times G_t}$ and $w_t = \frac{W_t}{G_1 \times \ldots \times G_t}$.

I rewrite the budget constraint (5) in terms of gross wealth inclusive of income (i.e., cash-on-hand), which is denoted by $X_{i,t} = (1+r_t)K_{i,t} + M_{i,t}$. The total economic resources available to household i in period t ($X_{i,t}$) are divided into consumption ($C_{i,t}$) and savings for the next period ($K_{i,t+1}$). In small letters due to scaling with $\{G_t\}$, $x_{i,t}$ evolves as follows:

$$x_{i,t+1} = \frac{R_{t+1}}{G_{t+1}}(x_{i,t} - c_{i,t}) + m_{i,t+1}, \tag{6}$$

where $R_t = 1 + r_t$. Specification (6) is consistent with standard consumption and savings decision problems (see, e.g., Zeldes, 1989; Deaton, 1991; Carroll and Kimball, 1996). Finally, the (scaled) borrowing limit is denoted by η:[3]

$$x_{i,t+1} \geq -\eta. \tag{7}$$

Information structure. The economy is assumed to converge in the long run to a balanced growth path equilibrium with a constant growth rate: $g_t \to g$ as $t \to \infty$. Economic agents have perfect foresight regarding the aggregate variables, including the growth path of productivity $\{g_t\}$. However, households are exposed to uncertainty through their idiosyncratic income risk $\varepsilon_{i,t}$ and death shocks. This specification enables me to emphasize the macroeconomic implications of household heterogeneity without oversimplifying household heterogeneity and sacrificing the tractability of the model.

Optimality conditions. Household consumption and savings decisions are characterized by two sufficient conditions for optimality, the consumption Euler equation and the transversality condition (see, e.g. Stokey, Lucas and Prescott, 1989). Under CRRA preferences, the consumption Euler equation is given by

$$1 \geq \mathbb{E}_t \left[\beta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} R_{t+1} \right], \tag{8}$$

where equality holds if the borrowing limit is not binding. When the financial market is complete, this Euler equation simplifies to $1 = \beta G_{t+1}^{-\gamma} R_{t+1}$. However, when households face uninsurable idiosyncratic income risk, consumption growth is not constant across households, and $\frac{C_{i,t+1}}{C_{i,t}}$ is not equal to G_{t+1} in general. The transversality condition is given by

$$\lim_{T\to\infty} \mathbb{E}_t \left[\beta^T (C_{i,T})^{-\gamma} X_{i,T} \right] = 0. \tag{9}$$

[3] Alternatively, the borrowing constraint can be expressed in terms of saving for the next period: $x_{i,t} - c_{i,t} \geq -\psi$ for a constant ψ. The results in this paper do not change when $\psi = \max_x \zeta(x+\eta)^\xi - x = (\zeta\xi)^{1/(1-\xi)}(\xi^{-1} - 1) - \eta$ for the parameters ζ and ξ introduced in Assumption 1. Note that the optimal consumption decision in Proposition 1, $\bar{c}(x_{i,t}) = \zeta(x_{i,t} + \eta)^\xi$, satisfies the constraint for all $x_{i,t} \geq -\eta$.

Idiosyncratic income risk. Because $m_{i,t} = w_t \varepsilon_{i,t}$ and each household takes w_t as given, specifying an exogenous idiosyncratic labor productivity process $\{\varepsilon_{i,t}\}$ is equivalent to characterizing the stochastic income process $\{m_{i,t}\}$. I impose the following assumption, which is judiciously chosen to yield tractable solutions to the household optimization problem in an incomplete financial market with only a risk-free asset.

Assumption 1 (Idiosyncratic income risk)**.** *Consider a random variable $z_{i,t}$ that governs idiosyncratic income risk. I assume that*

(EE1) *$z_{i,t}$ is independent across time and individuals, and*

(EE2) $1 = \mathbb{E}\left[\beta R_{t+1} G_{t+1}^{-\gamma} \exp(-\xi\gamma z_{i,t+1})\right]$ *for a constant $0 < \xi < 1$ and all t.*

I define an auxiliary exogenous process $\{\tilde{x}_{i,t}\}$ such that

(EE3) *$\tilde{x}_{i,0} = x_{i,0} > -\eta$, and*

(EE4) *$\tilde{x}_{i,t+1} + \eta = \exp(z_{i,t+1})(\tilde{x}_{i,t} + \eta)$ for all t.*

For all $t \geq 0$ and a constant $\zeta > 0$, the (scaled) income process $\{m_{i,t+1}\}$ is exogenously given by

(EE5) $m_{i,t+1}(\tilde{x}_{i,t}, z_{i,t+1}) = \frac{R_{t+1}}{G_{t+1}}\zeta\left(\tilde{x}_{i,t} + \eta\right)^{\xi} + \left[\exp(z_{i,t+1}) - \frac{R_{t+1}}{G_{t+1}}\right]\left(\tilde{x}_{i,t} + \eta\right) - \frac{G_{t+1} - R_{t+1}}{G_{t+1}}\eta.$

I assume further that

(TVC1) *$g_t \to g$, $\mu_{z,t} \to \mu_z$, $\sigma_{z,t} \to \sigma_z$, and $r_t \to r$ for some constants g, μ_z, σ_z, and r as $t \to \infty$,*

(TVC2) *$z_{i,t}$ is a Laplacian random variable with mean $\mu_{z,t}$ and variance $\sigma_{z,t}^2$ for all t; $z_{i,t} \sim \mathcal{L}(\mu_{z,t}, \sigma_{z,t})$, and*

(TVC3) *$\max\{\gamma_L, 0\} < \gamma < \gamma_U$ for some constants γ_L and γ_U that depend on $\mu_z, \sigma_z, \beta, \xi, g$, and η (see Appendix C for details).*

Assumption 1(EE1-5) specifies the individual income process $\{m_{i,t+1}\}$. This process is exogenous to household decisions and is determined by an auxiliary exogenous process $\{\tilde{x}_{i,t}\}$ (EE3-4) and idiosyncratic income shocks $\{z_{i,t}\}$ (EE1) that satisfy a moment condition (EE2). Assumption 1(TVC1-3) imposes a bound on the feasible values of the relative risk aversion coefficient γ. This bound depends on the balanced growth path that the economy converges to in the long run (TVC1) and the distributional family of the idiosyncratic income shocks $z_{i,t}$ (TVC2). I assume a Laplacian shock, which better matches the data than a Gaussian shock, as shown in Lee (2021).

Consumption function. Assumption 1 generates an environment that admits a highly tractable representation of the consumption–savings decision of each household.

Proposition 1 (Consumption function)**.** *Consider the following consumption function expressed in terms of scaled consumption ($c_{i,t}$) and of wealth inclusive of income ($x_{i,t}$):*

$$c_{i,t} = \bar{c}(x_{i,t}) = \zeta(x_{i,t} + \eta)^{\xi} \quad \textit{for} \quad x_{i,t} \geq -\eta, \tag{10}$$

where $\zeta > 0$ and $0 < \xi < 1$. Assumption 1(EE1-EE5) implies that this consumption function satisfies the Euler equation (8) with equality for all t. Furthermore, Assumption 1(TVC1-TVC3) is a sufficient condition for the transversality condition (9) to hold. Thus, the consumption function (10) solves the households' dynamic programming problem.

Individual consumption ($c_{i,t}$) is determined by both asset holdings ($k_{i,t}$) and income ($r_t k_{i,t} + m_{i,t}$) because wealth inclusive of income ($x_{i,t}$) is given by $(1+r_t)k_{i,t} + m_{i,t}$. Additionally, this highly tractable consumption function is characterized by only three parameters: ζ, ξ, and η reflect the scale, shape, and borrowing limit, respectively. Because $0 < \xi < 1$, the consumption function (10) is strictly concave in x, implying that rich households save more. This result is consistent with the empirical evidence in Dynan, Skinner and Zeldes (2004), Johnson, Parker and Souleles (2006), Parker et al. (2013), and Zidar (2019). Furthermore, cross-sectional relationships between consumption and wealth in the data are approximated remarkably well by this simple function (see Section 5.1).

This model differs from other models with uninsurable income fluctuations that have only numerical solutions. The important exceptions include Constantinides and Duffie (1996) and Heathcote, Storesletten and Violante (2014), which feature a tractable equilibrium with no trade among households. My model augments these seminal papers with a significant amount of asset trading among households in equilibrium. Furthermore, in this paper, the quantity of assets is non-trivial and equals the endogenously determined demand of capital used in production, whereas capital is not a factor of production in Constantinides and Duffie (1996) and Heathcote, Storesletten and Violante (2014). Other approaches to building tractable incomplete-market models utilize either linear or piecewise-linear consumption functions (e.g., Wang, 2003, 2007; Wen, 2015; Acharya and Dogra, 2020). In contrast, the consumption function (10) is strictly concave in x, even for rich households with large x.

In several incomplete market models, consumption functions are asymptotically linear in wealth (see, e.g., Ma and Toda, 2021, 2022). Note that $\frac{dc}{dx} = \frac{d \log c}{d \log x} \frac{c}{x}$. If $\frac{dc}{dx}$ is approximately constant for large x as in these models and $\frac{c}{x}$ decreases in x as the rich save more, the elasticity of consumption $\left(\frac{d \log c}{d \log x}\right)$ should vary accordingly with x. In contrast, the consumption function (10) features a constant elasticity with respect to wealth augmented with η. This isoelastic functional form results from a specific income process in Assumption 1 and is consistent with the empirical evidence in Section 5.1.

Cross-sectional distributions of wealth. Building on the optimal consumption and savings decisions at the household level, I specify the cross-sectional distribution of wealth inclusive of income $x_{i,t}$. By imposing certain assumptions, I induce simple, parameterized wealth distributions based on three parameters. Furthermore, this model replicates the Pareto right tails and rising wealth inequality observed in recent decades by using different parameter values.

I start with the law of motion for household consumption and wealth. Under the optimal consumption–savings decision in Proposition 1 and conditional on survival, individual consumption and wealth have random growth processes driven by the idiosyncratic income shocks $\{z_{i,t}\}$.

Proposition 2 (Dynamics of individual consumption and wealth)**.** *When Assumption 1 holds and agent i does not die in period $t+1$, the following are true:*

(1) $x_{i,t+1} + \eta = \exp(z_{i,t+1})(x_{i,t} + \eta)$ *for all* t.

(2) $x_{i,t} = \tilde{x}_{i,t} > -\eta$ *for all* t.

(3) $c_{i,t+1} = \exp(\xi z_{i,t+1}) c_{i,t}$ *for all* t.

Wealth inclusive of income (augmented with the borrowing limit) randomly grows at the individual level. Furthermore, the auxiliary exogenous variable $\tilde{x}_{i,t}$ coincides with $x_{i,t}$, and the borrowing constraint does not bind for all t. Finally, given the consumption function (10), log consumption also has a random walk process, which is less volatile than the log wealth process because $\xi < 1$.

Let $a_{i,t}$ denote the logarithm of wealth augmented with income and the borrowing limit (and scaled by growth rates):

$$a_{i,t} = \log\left(x_{i,t} + \eta\right).$$

The random growth results in Proposition 2 imply that $a_{i,t+1} = a_{i,t} + z_{i,t+1}$ conditional on surviving. An agent may die with probability p_d after receiving income $r_{t+1}k_{i,t+1} + w_{t+1}\epsilon_{i,t+1}$ but before making consumption–savings decisions. The government confiscates these accidental bequests $x_{i,t+1}$ and redistributes them to the agents newly born in each period. The population mass of the newly born agents is p_d, and the logarithm of their random endowments augmented with the borrowing limit is denoted by $n_{i,t+1}$. The distribution of $n_{i,t+1}$ reflects nonmodeled redistribution mechanisms, such as estate taxes, public education, and social insurance, and the stochastic components of intergenerational transmission. Given i.i.d. death shocks $d_{i,t} \sim \text{Bernoulli}(p_d)$, the cross-sectional distribution of wealth evolves as follows:

$$a_{i,t+1} = (1 - d_{i,t+1})(a_{i,t} + z_{i,t+1}) + d_{i,t+1}n_{i,t+1}. \tag{11}$$

In terms of probability density functions (pdfs), Equation (11) yields:

$$f_{a,t+1} = (1 - p_d)f_{a+z',t+1} + p_d f_{n,t+1}, \tag{12}$$

where the pdfs of $a_{i,t+1}$, $a_{i,t} + z_{i,t+1}$, and $n_{i,t+1}$ are denoted by $f_{a,t+1}$, $f_{a+z',t+1}$, and $f_{n,t+1}$, respectively.

The following assumptions induce a Laplace distribution of $a_{i,t}$ in every period.

Assumption 2 (Cross-sectional distribution of wealth)**.** *I assume that*

(1) $a_{i,t} \sim \mathcal{L}(\mu_{a,t}, \sigma^2_{a,t})$,

(2) $z_{i,t+1} \sim \mathcal{L}(\mu_{z,t+1}, \sigma^2_{z,t+1})$ *is independent of* $a_{i,t}$*, and*

(3) $\sqrt{2} > \sigma_{a,t} > \sigma_{z,t+1} > 0.$

Let $a_{i,t+1}$ *have a mean of* $\mu_{a,t+1}$ *and variance* $\sigma^2_{a,t+1}$*. I assume further that*

(4) $\sigma_{a,t} \leq \sigma_{a,t+1}$*, and*

(5) $p_d > 1 - \frac{1}{\mathcal{C}_{t+1}}$, *where* $\mathcal{C}_{t+1} \equiv \frac{\sigma_{a,t}\sigma_{a,t+1}}{\sigma_{a,t}^2 - \sigma_{z,t+1}^2} \exp\left(\frac{|\mu_{z,t+1}| - \mu_{a,t+1} + \mu_{a,t}}{\sigma_{a,t}/\sqrt{2}}\right) \geq 1.$

I start with a log Laplace distribution of wealth in period t (Assumption 2 (1)). For the idiosyncratic income shock $z_{i,t+1}$, which is independent of $a_{i,t}$ (Assumption 2 (2)), the cross-sectional dispersion of log wealth is larger than the period-by-period idiosyncratic risk (Assumption 2 (3)). Furthermore, $\sqrt{2} > \sigma_a$ is assumed to make aggregate wealth finite. Assumption 2 (4) guarantees that this framework can be used to investigate the macroeconomic implications of (weakly) rising wealth inequality. Finally, the probability of death p_d is not too low (Assumption 2 (5)). These conditions are sufficient to induce a log Laplace wealth distribution in period $t+1$ with a carefully chosen pdf $f_{n,t+1}$.

Proposition 3 (Log Laplace distributions of wealth)**.** *Assumption 2 implies that there exists a well-defined pdf* $f_{n,t+1}$ *of the random endowment* $n_{i,t+1}$ *for the newly born agents such that*

$$a_{i,t+1} \sim \mathcal{L}(\mu_{a,t+1}, \sigma_{a,t+1}^2). \tag{13}$$

Thus, under a judiciously calibrated $\{f_{n,t}\}$, this model generates a highly tractable cross-sectional distribution of wealth $\log(x_{i,t}+\eta) = a_{i,t} \sim \mathcal{L}(\mu_a, \sigma_{a,t}^2)$ in every period. Furthermore, this cross-sectional distribution of wealth is characterized by only three parameters: the borrowing limit η, the central tendency of log wealth $\mu_{a,t}$, and the measure of dispersion or inequality $\sigma_{a,t}$. Note that rising wealth inequality can be modeled by an increase in $\sigma_{a,t}$.

In summary, the household side of this economy admits a tractable nonlinear consumption function (10) and parametric cross-sectional distributions of wealth (13). Thus, it is possible to summarize the equilibrium conditions from a continuum of heterogeneous households with only a few analytically tractable equations. Because firm R&D investments depend on the market size, which affects the profit gained from the improved production technology, deriving an equation for aggregate demand intensity and relating it to the distributional parameters of the economy plays a crucial role in the subsequent analysis.

4 Analytical characterization of the steady state

This section characterizes the general equilibrium of the model. Using analytical expressions, I illustrate how demand intensity, defined as the ratio between aggregate consumption and wealth inclusive of national income, connects productivity growth with wealth inequality.

For clarity of exposition, this section focuses on a balanced growth path equilibrium. In Section 5, quantitative analysis is used to show that similar intuition and results are valid for the transition dynamics.

4.1 Demand intensity and productivity growth

This section illustrates how demand intensity relates to the major supply-side components of the economy. Specifically, demand intensity affects market size, R&D spending, and endogenously determined productivity growth rates. Using the equilibrium conditions from the firm side of the economy presented in Section 3.1, I show how demand intensity determines productivity growth rates and other major macroeconomic variables in this model. Because demand intensity summarizes the consumption–savings decisions of heterogeneous households, it connects the demand side with the supply side of the economy in a succinct manner.[4]

Let C_t, K_t, and Y_t be aggregate consumption, capital, and output, respectively. Aggregate wealth inclusive of income is denoted by X_t, where X_t includes capital and national income net of capital depreciation:

$$X_t = (1-\delta)K_t + Y_t.$$

Reflecting the consumption–savings decisions of households, a portion of X_t is consumed (C_t), and the remainder is saved (K_{t+1}). Then, demand intensity is defined as the ratio between aggregate consumption and wealth inclusive of national income:

$$s_C = \frac{C_t}{X_t}.$$

Similarly, I express other variables that grow in terms relative to X_t, e.g., $s_Y = \frac{Y_t}{X_t}$, $s_K = \frac{K_t}{X_t}$, $s_W = \frac{W_t}{X_t}$, and $s_A = \frac{A_t}{X_t}$. On the balanced growth path, these ratios are constant.

Proposition 4 (Endogenous growth)**.** *On a balanced growth path with symmetric firms, the*

[4]The discussion below focuses on the endogenous determination of productivity growth rates in relation to demand intensity. The results for other macroeconomic variables echo the findings in Lee (2021) for a neoclassical model with exogenous growth.

first-order conditions for firms' optimization problem imply that

$$r = \mathcal{M}^{-1}(1-\alpha)\frac{s_Y}{s_K} - \delta, \tag{14}$$

$$r = \theta L_Y + g, \tag{15}$$

where $\mathcal{M} = \frac{\omega}{\omega-1} > 1$ *is the gross markup. Given* $s_Y(s_C, g) = s_C + \frac{\delta+g}{1+g}(1-s_C)$*,* $s_K(s_C, g) = \frac{1-s_C}{1+g}$*,* $L_Y(s_C, s_A, g) = \left(\frac{s_Y(s_C,g)}{s_A^\alpha s_K(s_C,g)^{1-\alpha}}\right)^{1/\alpha}$*, and* s_A*, the following equation relates* g *to* s_C*:*

$$\mathcal{M}^{-1}(1-\alpha)\frac{s_Y(s_C, g)}{s_K(s_C, g)} - \delta = \theta L_Y(s_C, s_A, g) + g. \tag{16}$$

Equation (14) characterizes the optimal accumulation of capital ($K_{j,t}$). The first-order conditions with respect to R&D ($L_{A,j,t}$ and $A_{j,t+1}$) and the labor input into production ($L_{Y,j,t}$) yield Equation (15).[5] Note that the dependence of s_Y, s_K, and L_Y on g is made explicit in Equation (16) because g is also endogenously determined. To implicitly define $g(s_C, s_A)$ using Equation (16), I first show that a unique g that satisfies Equation (16) exists.

Proposition 5 (Unique existence of g)**.** *Fix* $s_C \in (0, 1)$ *and* $s_A > 0$*. When* $\frac{1-\alpha}{\mathcal{M}(1-s_C)} < 1$*, there exists a unique* g *satisfying Equation (16).*

A sufficient condition for the above result is $s_C \leq \alpha$ because $\mathcal{M} > 1$. Given $\alpha \approx 0.6$ and $s_C \approx 0.2$, this sufficient condition is likely to hold for realistic parameter values. Therefore, $g(s_C, s_A)$ can be defined implicitly from Equation (16) for $s_C \in (0, \alpha]$. This result allows me to rewrite $s_Y(s_C, g)$ as $s_Y(s_C, s_A) \equiv s_Y(s_C, g(s_C, s_A))$. Similarly, other macroeconomic variables, such as s_K, L_Y, L_A, s_W, and r, can be written as a function of s_C and s_A only.[6]

Proposition 6 (Comparative statics of $g(s_C, s_A)$)**.** *Suppose that* $g(s_C, s_A)$ *is defined as above and* $\frac{1-\alpha}{\mathcal{M}(1-s_C)} < 1$*. Then,* $\frac{\partial g(s_C,s_A)}{\partial s_C} > 0$ *if and only if*

$$\mathcal{M}^{-1}(1+g) > \frac{1-s_C}{\alpha}(\chi - g) + \frac{(1-\delta)(1-s_C)^2}{\alpha(1-\alpha)}\frac{\chi - g}{1+g-(1-\delta)(1-s_C)}, \tag{17}$$

[5] Let $\lambda_{A,j,t}$ denote the Lagrange multiplier associated with the R&D equation (3). The first-order condition with respect to $A_{j,t+1}$ implies that $\lambda_{A,j,t}$ equals $\frac{1}{1+r_{t+1}}\left[\mathcal{M}^{-1}\frac{Y_{j,t+1}}{A_{j,t+1}} + \lambda_{A,j,t+1}\frac{A_{j,t+2}}{A_{j,t+1}}\right]$. Clearly, other things being equal, the demand $Y_{j,t+1}$ increases the shadow value of $A_{j,t+1}$ and, therefore, R&D investment.

[6] For s_Y, s_K, L_Y, and r, I use the expressions in Proposition 4. $s_{K'}(s_C, s_A)$ is given by $[1 + g(s_C, s_A)]s_K(s_C, s_A)$. Because $g = \theta L_A$, $L_A(s_C, s_A) = \theta^{-1}g(s_C, s_A)$. From the first-order condition for $L_{Y,j,t}$, I have $WL_Y = \mathcal{M}^{-1}\alpha Y$; therefore, $s_W(s_C, s_A) = \mathcal{M}^{-1}\alpha\frac{s_Y(s_C.s_A)}{L_Y(s_C,s_A)}$.

where $\chi = g + \frac{\theta}{s_A}(1-s_C)^{\frac{\alpha-1}{\alpha}}\frac{(g+\delta+(1-\delta)s_C)^{1/\alpha}}{1+g}$. *As a special case, when* $\alpha > s_C$, $\alpha > \frac{1}{2}$, *and* $g(s_C, s_A) > -\left[1 + \frac{(1-s_C)\mathcal{M}}{2\alpha-1}\right]^{-1}$, *the above condition holds for a sufficiently large* δ.

When $\frac{\partial g(s_C,s_A)}{\partial s_C} > 0$, a decrease in demand intensity (a smaller s_C) has negative supply-side effects on productivity (a lower g). Although the general sufficient condition for this result in Equation (17) is not highly tractable, it is not overly restrictive. At a minimum, the three inequality conditions for the special case with a large δ are likely to hold. Furthermore, Equation (17) is satisfied for the realistic parameter values used in this paper.

Intuitively, on a balanced growth path with lower consumption demand s_C, the market is smaller. Because of limited profit gains from an improved production technology, firms may have fewer incentives to spend on R&D. Concurrently, a smaller s_C and more savings s_K increase the wage rate s_W and decrease the real interest rate r. As hiring researchers becomes more expensive, spending on R&D is further deterred. In contrast, as firms discount future profits at a lower rate, additional R&D and improved productivity become more attractive. For the calibrated parameter values in this paper, the market size and wage effects dominate the real interest rate effect. In short, g decreases when s_C reduces.

This model implies that the limited consumption demand observed in the data (panel (d) in Figure 1) might have yielded substantial negative effects on supply-side trends. When financial markets are incomplete, this decrease in demand intensity can result from an increase in economic inequality. As shown in Section 3.2, uninsurable idiosyncratic income risk implies that the consumption of individual households is a concave function in wealth inclusive of income (see also Carroll and Kimball, 1996). Given this nonlinear consumption function, demand intensity depends on the distributional elements of the model.[7]

4.2 Demand intensity and wealth inequality

This section presents the relationship between demand intensity and wealth inequality. The tractable model of heterogeneous households presented in Section 3.2 allows for a straightfor-

[7] This prediction of the incomplete market model does not hold when the financial markets are complete. In a complete market economy, the Euler equation (8) simplifies to $R = G^\gamma/\beta$. In combination with Equation (14), an additional equilibrium condition is obtained: $\mathcal{M}^{-1}(1-\alpha)\frac{s_Y(s_C,g)}{s_K(s_C,g)} - \delta = \frac{(1+g)^\gamma}{\beta} - 1$. Then, this equation and Equation (16) jointly determine the equilibrium values of s_C and g, given s_A. Because s_C is fixed regardless of the cross-sectional allocation of wealth, the comparative static results in Proposition 6 are less relevant in this environment. However, if the market is incomplete, there exists a gap between individual consumption growth $\frac{C_{i,t+1}}{C_{i,t}}$ in the Euler equation (8) and the aggregate growth rate G_{t+1} because of the uninsurable idiosyncratic income shocks.

ward aggregation of individual consumption and wealth. Then, demand intensity s_C admits an analytical representation that clearly illustrates how wealth inequality affects aggregate consumption demand. As in Section 4.1, I focus on a balanced growth path equilibrium for exposition.

Propositions 1 and 3 imply that wealth inclusive of income has a log Laplace distribution and $c_i = \zeta(x_i + \eta)^\xi$. Using the properties of Laplace distributions, I obtain the following results.

Proposition 7 (Demand intensity, s_C)**.** *Suppose that $a_i = \log(x_i + \eta) \sim \mathcal{L}(\mu_a, \sigma_a^2)$ and $c_i = \bar{c}(x_i) = \zeta(x_i + \eta)^\xi$. Then,*

(1) *The right tail of wealth x_i follows a Pareto distribution.*

(2) *If $\sigma_a < \sqrt{2}$, aggregate wealth and consumption (scaled) are given by*

$$x = \mathbb{E}_i[\exp(a_i)] - \eta = \frac{\exp(\mu_a)}{1 - \frac{1}{2}\sigma_a^2} - \eta, \quad \textit{and}$$
$$c = \mathbb{E}_i[c_i] = \frac{\zeta \exp(\xi \mu_a)}{1 - \frac{1}{2}\xi^2\sigma_a^2}.$$

(3) *Suppose $\sigma_a < \sqrt{2}$. Given a mean-preserving spread in x_i induced by changes in μ_a and σ_a, $s_C = \frac{c}{x}$ decreases:*

$$\frac{ds_C}{d\sigma_a^2}|_x = -\mathcal{D}s_C < 0,$$

where $\mathcal{D} = \frac{1}{2}\frac{1+0.5\xi\sigma_a^2}{(1-0.5\xi^2\sigma_a^2)(1-0.5\sigma_a^2)}\xi(1-\xi) > 0.$

This simple model of the cross-sectional distribution of wealth features only three parameters (η, μ_a, and σ_a). Despite being simple, this model successfully replicates the stylized fact of a Pareto right tail in the distribution of wealth (Proposition 7 (1)), consistent with the empirical evidence in Saez and Zucman (2016). Furthermore, aggregate consumption and wealth (inclusive of income) admit analytically tractable representations, as shown in Proposition 7 (2). Thus, demand intensity $s_C = \frac{c}{x}$ is characterized by only five parameters (Proposition 7 (3)): the two consumption function parameters (ζ and ξ) and the three wealth distribution parameters (η, μ_a, and σ_a). To illustrate the effects of wealth inequality on demand intensity, I consider a mean-preserving spread of $\{x_i\}$. When σ_a^2 increases and

μ_a adjusts accordingly to keep aggregate wealth x constant, $\frac{ds_C}{d\sigma_a^2}|_x < 0$.[8] Intuitively, rising wealth inequality decreases aggregate consumption demand because rich households save more. Furthermore, the sensitivity of demand intensity to wealth inequality, captured by the coefficient $\mathcal{D}$, depends on the value of ξ and σ_a. Suppose, for illustrative purposes, that $\sigma_a = 1$ (see also Table 1). Then, as ξ changes from 0.45 to 0.55 (see Figure 3), $\mathcal{D}$ increases from 0.34 to 0.37.

Propositions 6 and 7 formalize the main insights from this model by connecting wealth inequality through aggregate consumption demand to productivity growth. Through its negative effects on demand intensity, an increase in wealth inequality can endogenously generate productivity slowdowns under incomplete financial markets.

4.3 A balanced growth path equilibrium

Building on the results from the previous sections, I characterize a balanced growth path equilibrium in this section. I specify parameters for idiosyncratic income risk (μ_z and σ_z) and the endowment distribution for the newly born agents (f_n) that are consistent with a log-Laplacian wealth distribution $a_i \sim \mathcal{L}(\mu_a, \sigma_a^2)$ in a balanced growth path equilibrium.

I start from a cross-sectional distribution of wealth inclusive of income, $\log(x_i + \eta) = a_i \sim \mathcal{L}(\mu_a, \sigma_a^2)$. Given the values of μ_a and σ_a, Proposition 7 determines s_C:

$$s_C = \frac{C_t}{X_t} = \frac{\zeta \exp(\xi \mu_a)}{1 - \frac{1}{2}\xi^2 \sigma_a^2} \Big/ \left[\frac{\exp(\mu_a)}{1 - \frac{1}{2}\sigma_a^2} - \eta \right].$$

This equation succinctly summarizes individual households' consumption–savings decisions at the aggregate level. Then, I use the equilibrium conditions from the firm side of the economy. Propositions 4 and 5 illustrate how to find the productivity growth rate g given s_C and s_A. Similarly, the other major macroeconomic variables, such as s_Y, s_K, s_W, r, L_Y, and L_A, can be fixed based on s_C and s_A (see footnote 6). Finally, L_M, which governs the per-period overhead labor cost, is calibrated to satisfy the zero-profit condition ($\Pi = 0$, Equation (4)) and prevent the entry of new firms.

To find a magnitude of the idiosyncratic income risk (μ_z and σ_z) and an endowment distribution for the newly born agents (f_n) that are consistent with the above macroeco-

[8] There is no trend-level change in the value of η estimated from the Panel Study of Income Dynamics (PSID) data (see Section 5.1). Thus, I concentrate on an increase in the dispersion parameter σ_a.

nomic environment in a general equilibrium, I turn to the consumption Euler equation (8), the labor market clearing condition, and the government's budget constraint regarding the redistribution of accidental bequests.

Given the real interest rate $r(s_C, s_A)$ and the growth rate $g(s_C, s_A)$, I obtain a moment condition for $z_{i,t} \sim \mathcal{L}(\mu_z, \sigma_z^2)$ from the Euler equation: $\mathbb{E}[\beta(1+r)(1+g)^{-\gamma} \exp(-\xi\gamma z_{i,t})] = 1$ (Assumption 1(EE2)). To clear the labor market, labor demand $L(s_C, s_A) = L_Y(s_C, s_A) + L_A(s_C, s_A) + L_M$ should equal labor supply in efficiency units, $\mathbb{E}_i[\varepsilon_{i,t}]$. With some algebra, it can be shown that this condition is equivalent to a moment condition $\mathbb{E}[\exp(z_{i,t})] = 1$. Finally, the government's budget clearing condition implies that the aggregate value of the accidental bequests is equal to the endowments of the newly born agents. Because death shocks are random, this condition is equivalent to $p_d\mathbb{E}[\exp(a_{i,t} + z_{i,t+1})] = p_d\mathbb{E}[\exp(n_{i,t+1})]$. Similarly, this equation simplifies to the same moment condition $\mathbb{E}[\exp(z_{i,t})] = 1$ as the labor market clearing condition. Note that the two moment conditions, $\mathbb{E}[\beta(1+r)(1+g)^{-\gamma} \exp(-\xi\gamma z_{i,t})] = 1$ and $\mathbb{E}[\exp(z_{i,t})] = 1$, are sufficient to determine the idiosyncratic income shock parameters (μ_z and σ_z).

What remains is to specify f_n in a manner such that the implied cross-sectional distribution of wealth f_a is consistent with this balanced growth path equilibrium. It straightforwardly follows from Equation (12) that it is sufficient to assume that $f_n = p_d^{-1}[f_a - (1-p_d)f_{a+z'}]$. As long as $p_d > 1 - \frac{1}{\mathcal{C}}$, where $\mathcal{C} = \frac{\sigma_a^2}{\sigma_a^2 - \sigma_z^2} \exp\left(\frac{|\mu_z|}{\sigma_a/\sqrt{2}}\right)$, this endowment distribution has a well-defined pdf (Proposition 3). I checked that this condition is satisfied for all the quantitative analyses in this paper. Similarly, the sufficient condition for the transversality condition, $\max\{\gamma_L, 0\} < \gamma < \gamma_U$ (Assumption 1 (TVC3)), is also satisfied.

5 Wealth inequality and productivity slowdowns

This section presents the quantitative analysis of the model. I compute the general equilibrium transition dynamics of the macroeconomic variables induced by a trend-level increase in idiosyncratic income risk and cross-sectional wealth inequality. Consistent with the theoretical prediction in Section 4, rising wealth inequality generates productivity slowdowns as well as other macroeconomic trends, such as low demand intensity, low real interest rates, and a high capital-to-net national income ratio. Comparing the different balanced growth path equilibria with the various levels of steady-state wealth inequality yields similar results.

5.1 Calibration

This section calibrates the model parameters. The results for the parameters on the household side of the model, such as the distributional parameters ($\mu_{a,t}$, $\sigma_{a,t}$, and η) and the consumption function parameters (ζ and ξ), are borrowed from Lee (2021). Here, I recapitulate the discussion in Lee (2021) for completeness. The R&D productivity parameter θ is chosen by matching the trend-level growth rate for the US economy at the beginning of the sample period. Other parameters are standard.

I start with the wealth distribution parameters. I compute the quintile shares of wealth inclusive of income ($x_{i,t}$) in the Panel Study of Income Dynamics (PSID) data and estimate $\mu_{a,t}$, $\sigma_{a,t}$, and η_t by matching these moments, where $\log(x_{i,t} + \eta_t) = a_{i,t} \sim \mathcal{L}(\mu_{a,t}, \sigma_{a,t}^2)$. In doing so, I conduct two exercises. First, I calibrate the dispersion parameter $\sigma_{a,t}$ and the borrowing limit η_t in each period without any restrictions to match the empirical quintile shares. Second, I impose a linear time-trend $\sigma_{a,t} = \kappa_0 + \kappa_1 t$ on the dispersion parameter and assume a time-invariant borrowing limit η (scaled) to focus on slow-moving changes in wealth inequality since the 1980s.[9] In this case, the three parameters, κ_0, κ_1, and η, are calibrated by jointly matching the quintile shares in all PSID waves during the sample period. In all cases, $\mu_{a,t}$ is chosen by normalizing aggregate wealth (scaled) in each period to one ($\mathbb{E}_i[x_{i,t}] = 1$). For details on the PSID data, see Appendix B.2.

Columns (1)-(5) in Table 1 show the empirical and model-implied quintile shares of wealth inclusive of income in selected years. In the data, the top quintile shares have substantially increased since the 1980s. Note that this secular increase in wealth inequality can be replicated by the model using the calibrated time series of the distributional parameters. Even the parsimonious model with an upward linear trend in $\sigma_{a,t}$ matches the cross-sectional wealth data in different years remarkably well. Finally, there exist no clear trend-level changes in the estimated η_t for each PSID wave.

Suppose that in 1983, the economy is on a balanced growth path described by the parameters in Table 1 ($\sigma_a = 0.93$, $\mu_a = -0.37$, and $\eta = 0.21$). Using the moment conditions in Section 4.3, I obtain $\sigma_z = 0.11$ and $\mu_z = -0.006$ for this balanced growth path equilibrium. Note that these realistic parameters satisfy Assumption 1 (3) ($\sqrt{2} > \sigma_a > \sigma_z$). Verifying Assumption 1 (5) ($p_d > 1 - \frac{1}{C}$) is also straightforward. Finally, Figure 2 depicts the model pdfs (f_a, $f_{a+z'}$, and $f_{n'}$) determined by the calibrated distributional parameters. Clearly,

[9] The 1984 wave is the first PSID wave with wealth data.

Table 1: Quintile shares of wealth inclusive of income in the data and the model

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Quintile Shares of Wealth Inclusive of Income (x, %)							
		1st	2nd	3rd	4th	5th	μ_a	σ_a	η
Data.	*PSID 1984 wave*	1	5	9	17	67			
Model.	*Flexible* $\sigma_{a,t}$ *and* η_t	0	6	10	16	67	-0.29	0.89	0.24
Model.	$\sigma_{a,t} = \kappa_0 + \kappa_1 t$ and $\eta_t = \eta$	0	6	10	16	69	-0.37	0.93	0.21
Data.	*PSID 2001 wave*	1	4	7	15	73			
Model.	*Flexible* $\sigma_{a,t}$ *and* η_t	0	5	8	14	73	-0.51	1.00	0.19
Model.	$\sigma_{a,t} = \kappa_0 + \kappa_1 t$ and $\eta_t = \eta$	0	4	8	14	74	-0.50	1.00	0.21
Data.	*PSID 2015 wave*	0	2	5	12	80			
Model.	*Flexible* $\sigma_{a,t}$ *and* η_t	-1	3	6	11	80	-0.68	1.07	0.19
Model.	$\sigma_{a,t} = \kappa_0 + \kappa_1 t$ and $\eta_t = \eta$	-1	3	7	12	79	-0.62	1.05	0.21

Notes: Columns (1)-(5) in this table show the quintile shares of wealth inclusive of income in selected PSID waves. The log-Laplacian model parameters are calibrated by matching the quintile shares of wealth. For the flexible parameter case, I change the dispersion parameter $\sigma_{a,t}$ and the borrowing limit η_t in each period without any restrictions. For the second case, I impose a linear time-trend $\sigma_{a,t} = \kappa_0 + \kappa_1 t$ on the dispersion parameter and assume a time-invariant borrowing limit η (scaled). In all cases, $\mu_{a,t}$ is determined by normalizing aggregate wealth to one in each period.

the wealth of the newly born agents (f_n) is more concentrated around the center of the distribution than the wealth of the entire population (f_a) is.

Next, I turn to the shape (ξ) and scale (ζ) parameters of the consumption function (10). Because $\log(c_{i,t}) = \text{constant } + \xi \log(x_{i,t} + \eta)$, the shape parameter ξ can be estimated by regressing $\log(c_{i,t})$ on $\log(x_{i,t} + \eta)$. The left panel of Figure 3 shows a binned scatter plot of $\log(c_{i,t})$ and $\log(x_{i,t} + \eta)$ and a regression line based on the 2003 PSID wave. I use the same $\eta = 0.21$ in Table 1. Note that, despite being simple, the tractable consumption function (10) captures the salient empirical relationships between wealth and consumption reasonably well. The right panel shows the ξ estimated from the different PSID waves with the 95% confidence intervals. In all cases, the estimates are similar and approximately 0.5. Thus, I calibrate ξ at 0.5.

For the scale parameter ζ, I compare the model-implied demand intensity $s_{C,t} = \frac{C_t}{X_t}$ with the corresponding empirical measure from the beginning of the sample period. By

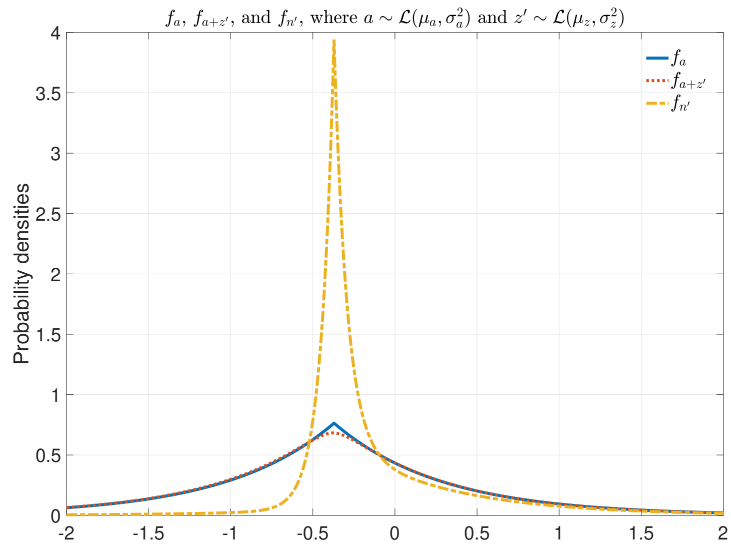

Figure 2: Cross-sectional distributions of wealth

Notes: This figure shows the model pdfs (f_a, $f_{a+z'}$, and $f_{n'}$) from Equation (12) under the assumption that the economy is in a balanced growth path equilibrium in 1983. I use the following parameters: $\sigma_a = 0.93$, $\mu_a = -0.37$, $\eta = 0.21$, $\sigma_z = 0.11$, $\mu_z = -0.006$, and $p_d = 0.025$.

fitting a linear time trend to the post-1980 data to focus on slow-moving changes, I obtain $s_{C,1983} = 19.5\%$ (see Figure 5 (a)). I adjust ζ to match the model-induced $s_{C,1983}$ of 19.5%, given the calibrated $\mu_{a,1983}$, $\sigma_{a,1983}$, η, and ξ.

The R&D productivity parameter θ is pinned down in the following manner. I first normalize s_A to one. Given $s_{C,1983}$ and the similarly estimated trend-level GDP growth rate g_{1983} of the US economy (see Figure 5 (b)), I can determine r and L_Y using the equations in Proposition 4. Then, Equation (15) is used to find a consistent value for θ.

In short, the model is calibrated to resemble the US economy in the early 1980s. By doing so, I can evaluate the quantitative importance of increasing wealth inequality for productivity slowdowns and other macroeconomic changes that started to emerge circa 1980.

Other parameters are standard: $\beta = 0.96$, $\gamma = 2$, $s_A = 1$, $p_d = 1/40$, $\delta = 0.06$, and $\omega = 6$. Thus, the steady-state markup, $\frac{1}{\omega-1}$, is 20 percent, and α is selected by setting the capital income share, $\frac{\omega-1}{\omega}(1-\alpha)$, at 40 percent. These values are chosen following Rotemberg and Woodford (1997) and Rognlie (2016, Table 4), respectively. As in Carroll et al. (2017), the average working life is 40 years. The depreciation rate of the physical capital stock is 6 percent (Nadiri and Prucha, 1996, Table II).

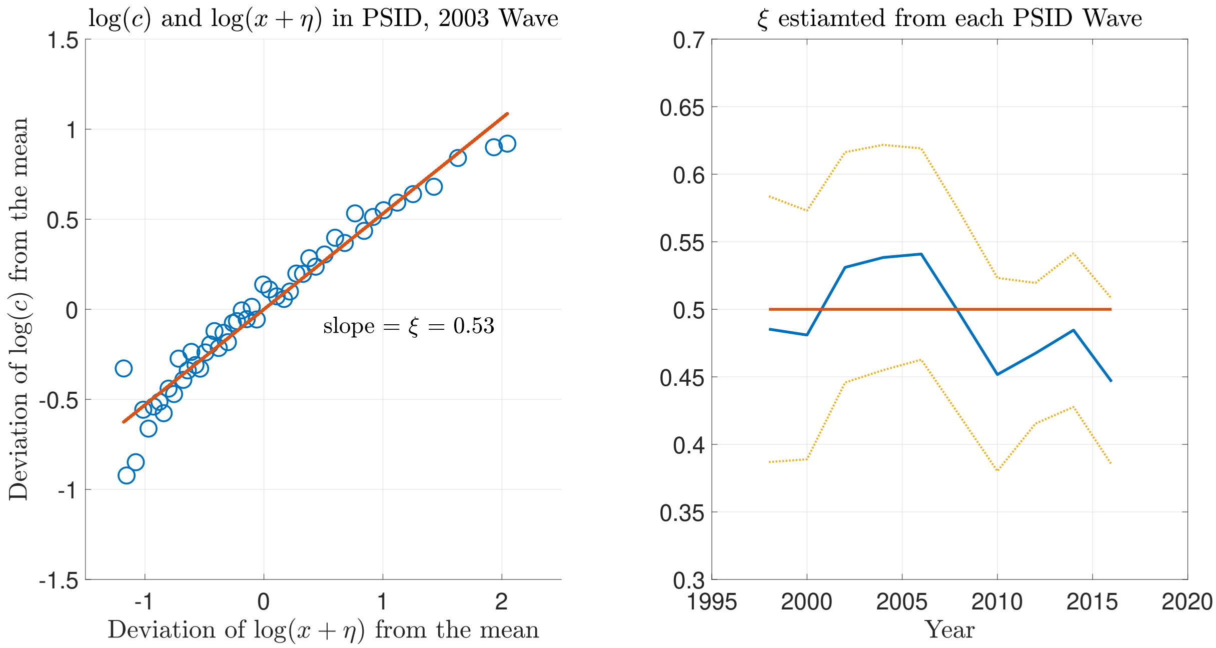

Figure 3: Consumption function in the PSID

Notes: The left panel shows a binned scatter plot of $\log(c_{i,t})$ and $\log(x_{i,t}+\eta)$ based on the PSID 2003 wave with $\eta = 0.21$. The right panel illustrates the estimated slope coefficient ξ with the 95% confidence intervals using each PSID wave and the calibrated value of 0.5.

5.2 Transition dynamics

This section quantitatively investigates the transition of the US economy from a low-inequality economy in the early 1980s to a high-inequality economy in the late 2020s. I examine the dynamics of major macroeconomic variables, such as productivity growth rates, demand intensity, real interest rates, and capital-to-income ratios, through the lens of the model. I document that rising wealth inequality, induced by an increase in idiosyncratic income risk and income inequality, generates substantial secular variation in these macroeconomic variables. Thus, I conclude that the distributional factors have significant macroeconomic implications for long-run growth as well as short-run fluctuations (see, e.g., Coibion et al., 2017; Kaplan and Violante, 2018).

I consider three cases with different equilibrium concepts. First, I calculate the general equilibrium transition dynamics. The transition starts from the low-inequality balanced growth path equilibrium in 1983. Then, in 1984, economic agents realize that wealth inequality $\sigma_{a,t}$ will exhibit a previously unanticipated gradual increase until 2019 and then stay at its 2019 level thereafter. Since 1984, economic agents have had perfect foresight regarding the corresponding dynamics of the macroeconomic variables, such as $\mu_{z,t}$, $\sigma_{z,t}$, $\mu_{a,t}$, and $f_{n,t}$. In 1984, the overhead labor cost L_M also adjusts to its value in the posttransition balanced growth path equilibrium with $\sigma_{a,2019}$ and $\mu_{a,2019}$. For this scenario, I assume that there exists a parsimonious time trend of $\sigma_{a,t} = \kappa_0 + \kappa_1 t$ with the time-invariant η in Table 1. As a benchmark, I consider a growth path such that X_t and A_t grow at the endogenously determined time-varying rate of g_t.[10]

The second case compares balanced growth path equilibria with different levels of wealth inequality. For the calibrated $\sigma_{a,t}$ and η_t based on each PSID wave, I compute the corresponding balanced growth path equilibrium. In this case, I use the wealth distribution parameters estimated without any restrictions.

Finally, I investigate partial equilibrium dynamics. I fit a linear time trend to the post-1980 observations of demand intensity $s_{C,t}$. Then, I input this secular decrease in s_C from $s_{C,1983}$ to $s_{C,2019}$ into the equilibrium conditions for the supply side to derive transition dynamics consistent with the exogenously assumed path of demand intensity. Because the household side is absent, this analysis is a partial equilibrium analysis.

Panels (a)-(c) in Figure 5 show the top wealth share, the standard deviation of log wealth, and the mean of log wealth, respectively, where $a_{i,t} = \log(x_{i,t} + \eta_t) \sim \mathcal{L}(\mu_{a,t}, \sigma^2_{a,t})$. The empirical top quintile shares estimated from the PSID are almost indistinguishable from the squares in panel (a), which represent the balanced growth path equilibria with different levels of wealth inequality. Additionally, with a gradual increase in $\sigma_{a,t}$, the transition path (the solid line) captures the increasing concentration of wealth at the top since the 1980s. This increase in wealth inequality is induced by an upward trend in $\sigma_{a,t}$ as shown in panel (b). Finally, given an increasing $\sigma_{a,t}$, preserving mean wealth $x_t = \frac{\exp(\mu_{a,t})}{1-0.5\sigma^2_{a,t}} - \eta_t$ through a mean-preserving spread requires a decrease in $\mu_{a,t}$ (panel (c)). The idiosyncratic income risk parameters $\sigma_{z,t}$ and $\mu_{z,t}$ are depicted in panels (d) and (e), where $z_{i,t} \sim \mathcal{L}(\mu_{z,t}, \sigma^2_{z,t})$. As is common in Bewley-Huggett-Aiyagari models, a larger income risk $\sigma_{z,t}$ is necessary to induce increased wealth inequality (panel (d)). Then, the labor market clearing condition $\mathbb{E}[\exp(z_{i,t})] = \frac{\exp(\mu_{z,t})}{1-0.5\sigma^2_{z,t}} = 1$ implies a concurrent decrease in $\mu_{z,t}$ as shown in panel (e). Finally, panel (f) illustrates the mode of $f_{n,t}$. As shown in Figure 2, the endowment of the newly born agents $f_{n,t}$ is highly concentrated around the center of the wealth distribution $f_{a,t}$. Thus, as $\mu_{a,t}$ decreases (panel (c)), the mode of $f_{n,t}$ similarly shifts leftward.

Figure 5 shows the main results from the quantitative analysis of the macroeconomic

[10] For details on the solution method, see Appendix D.1.

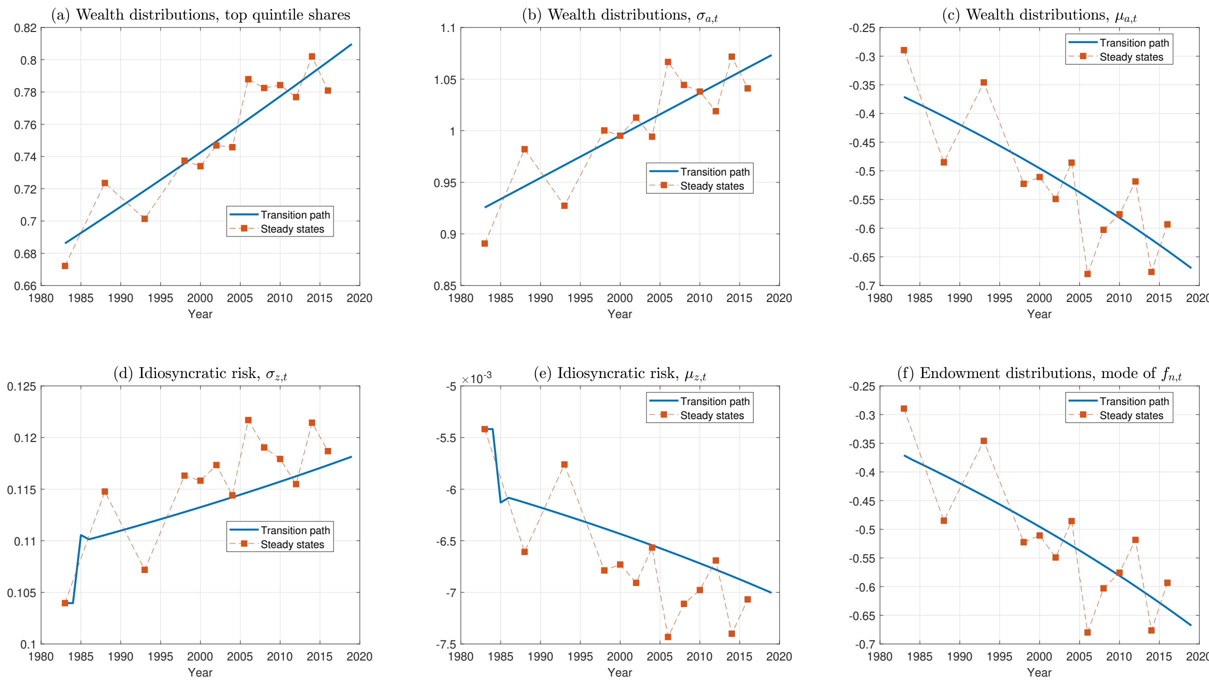

Figure 4: Distributional parameters

Notes: The general equilibrium transition path is shown by the solid lines. The balanced growth path equilibria with different levels of wealth inequality are depicted by the dashed lines with squares.

dynamics. Accompanied by the secular changes in the distributional factors shown in Figure 4, the aggregate consumption-to-wealth ratio (demand intensity) $s_{C,t}$ decreases (panel (a)), productivity growth rates decrease (panel (b)), real interest rates decrease (panel (c)), and the aggregate capital-to-net national income ratio increases (panel (d)). The magnitudes of these variations are substantial and comparable to the trend-level changes in the data since the 1980s. Thus, these results imply that rising wealth inequality can significantly affect macroeconomic trends through its effects on demand intensity $s_{C,t}$.[11]

Specifically, panel (a) illustrates the demand intensity, defined as the consumption-to-wealth (inclusive of income) ratio $s_{C,t} = \frac{C_t}{X_t}$. On the general equilibrium transition path (the solid line), $s_{C,t}$ decreases by 2 p.p. between 1983 and 2019. Because wealth inequality ($\sigma_{a,t}$) increases during transition periods and the consumption function (10) is concave, demand intensity decreases, consistent with the downward trend in the empirical time series (the dotted line) and theoretical predictions in Proposition 7 (3). When a linear time trend in the empirical measure of $s_{C,t}$ is directly estimated for the partial equilibrium analysis,

[11] For details on the empirical time series, see Appendix B.3.

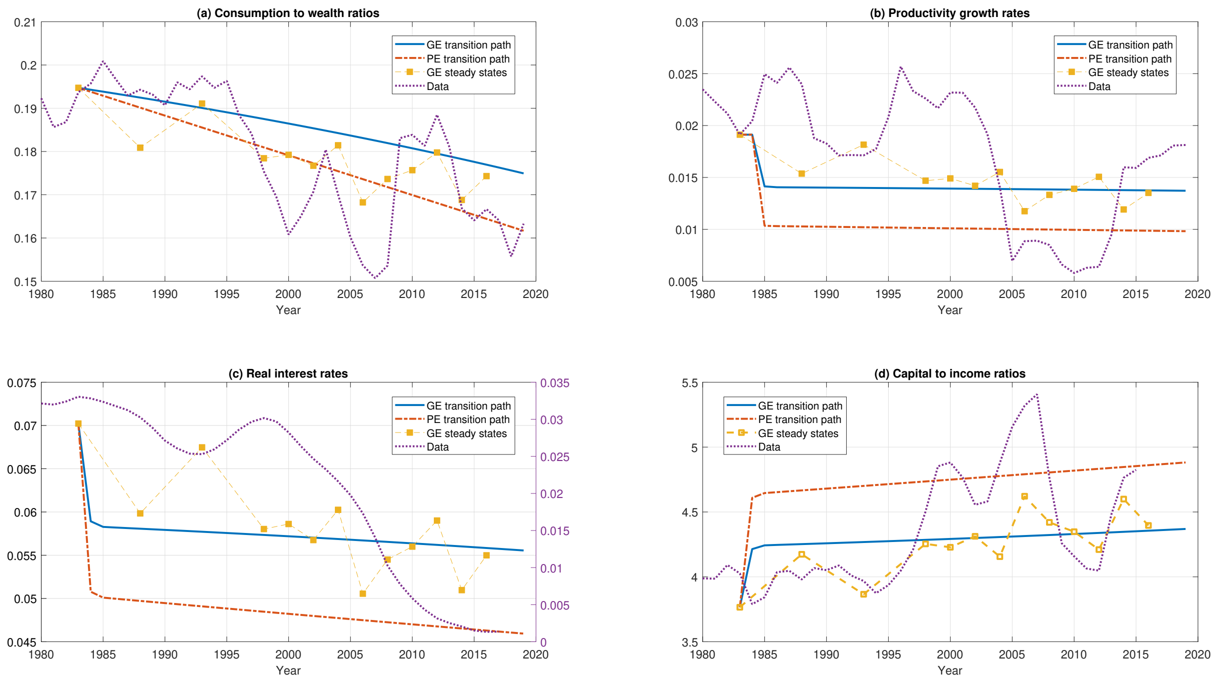

Figure 5: Macroeconomic trends in the model and the data

Notes: The general equilibrium transition path is shown by the solid lines. The partial equilibrium transition path with an exogenous decrease in $s_{C,t}$ is depicted by the dash-dotted lines. The balanced growth path equilibria with different levels of wealth inequality based on each PSID wave are illustrated by the dashed lines with squares. The dotted lines represent the empirical moments.

$s_{C,t}$ further decreases to 16.2% from 19.5% during the same periods (the dash-dotted line). Comparing the different balanced growth paths (the dashed line with squares) yields similar results. Thus, rising inequality and the concavity of the consumption function in wealth inclusive of income explain approximately two-thirds of the trend-level decrease in demand intensity in the data.

The limited demand and the small market size disincentivize firms from spending on R&D. Thus, as shown in Proposition 6 and panel (b), productivity growth slows endogenously and substantially. The transition to a high-inequality economy starts in 1984. As $L_{A,1984}$ decreases, $g_{1985} = \theta L_{A,1984}$ decreases accordingly. Subsequently, on the general equilibrium transition path, g_t gradually decreases and converges to 1.4%. Similar to demand intensity, shown in panel (a), the balanced growth paths with different levels of wealth inequality feature comparable growth slowdowns. In the partial equilibrium analysis, $s_{C,t}$ and g_t decrease more than along the general equilibrium path, and the posttransition growth rate is 1%. Thus, wealth inequality has significant growth implications, and this model prediction is consistent with the empirical time series of economic growth rates (see also Fernald, 2015; Antolin-Diaz, Drechsel and Petrella, 2017).

These secular trends in demand intensity and productivity growth are further related to other macroeconomic trends. Panel (c) depicts the model-implied r_t (left axis) and its empirical estimates (right axis). I obtain the following result for r_t from the Euler equation (8) (see also Assumption 1(EE2)):

$$r_{2019} - r_{1983} \approx \gamma(g_{2019} - g_{1983}) + \gamma\xi(\mu_{z,2019} - \mu_{z,1983}) - \frac{1}{2}(\xi\gamma)^2(\sigma^2_{z,2019} - \sigma^2_{z,1983}). \tag{18}$$

Along the general equilibrium transition path, the three terms on the right-hand side in Equation (18) equal 1.1 p.p., 0.2 p.p., and 0.2 p.p., respectively. Thus, the modestly intensified idiosyncratic income risk that underlies rising wealth inequality contributes substantially to low real interest rates. Additionally, this contribution occurs largely through the general equilibrium effects on demand intensity, R&D incentives, and productivity growth rates. The results from different balanced growth path equilibria are comparable with the decreases in r_t from the general equilibrium transition dynamics. Finally, the partial equilibrium analysis predicts a larger decrease in r_t based on a larger decrease in $s_{C,t}$.

Panel (d) illustrates the capital-to-net national income ratios $\frac{K_t}{Y_t - \delta K_t}$. The upward trend in the data documented by Piketty and Zucman (2014) is replicated by the model simulation results. Because $C_t + K_{t+1} = X_t$, a decrease in $s_{C,t}$ implies relatively more capital and, therefore, a higher value of $\frac{K_t}{Y_t - \delta K_t}$ in the model.

In summary, distributional factors are relevant for long-run macroeconomic dynamics. The model simulation results imply that rising wealth inequality in the US since the 1980s has been a major source of productivity slowdowns as well as other salient macroeconomic trends, such as low demand intensity, low real interest rates, and a high capital-to-income ratio.

5.3 Welfare implications of wealth inequality

This section assesses the welfare implications of rising wealth inequality. For this purpose, I define the following social welfare function for a constant λ:

$$SW\left(\{\lambda C_{i,t}\}\right) = \sum_t \beta^t \left[\int \frac{(\lambda C_{i,t})^{1-\gamma} - 1}{1-\gamma}\, di\right]. \tag{19}$$

The scaling by growth rates ($c_{i,t} = \frac{C_{i,t}}{G_1 \times \ldots \times G_t}$), the (scaled) consumption function (10) ($c_{i,t} = \zeta \exp(\xi a_{i,t})$), and the cross-sectional distribution of wealth (13) ($a_{i,t} \sim \mathcal{L}(\mu_{a,t}, \sigma^2_{a,t})$) allow the social welfare function $SW(\{\lambda C_{i,t}\})$ to be parameterized by λ, $\{g_t\}$, and $\{\mu_{a,t}, \sigma_{a,t}\}$.

I compare the balanced growth path equilibrium with low inequality with the general equilibrium transition path towards a high-inequality economy. The total social welfare cost of this increase in wealth inequality is summarized by λ^{total} in the following equation:

$$SW(\lambda = 1, g_{1983}, \mu_{a,1983}, \sigma_{a,1983}) = SW(\lambda^{total}, \{g_t\}, \{\mu_{a,t}, \sigma_{a,t}\}).$$

λ^{total} reflects the proportional increase in individual consumption on the transition path required to keep social welfare unchanged from that in the pretransition balanced growth path equilibrium. This cost is determined by two factors: productivity slowdowns $\{g_t\}$ and rising wealth inequality $\{\mu_{a,t}, \sigma_{a,t}\}$. Thus, I decompose λ^{total} into λ^{growth} and λ^{wealth}, where λ^{growth} reflects only the cost of productivity slowdowns and λ^{wealth} captures the increase in wealth inequality. Additionally, the increase in wealth inequality affects aggregate consumption (demand intensity) and consumption inequality. Thus, λ^{wealth} is further decomposed into the cost of decreasing average consumption relative to wealth (λ^{s_C}) and increasing consumption inequality ($\lambda^{consumption}$).

Using the general equilibrium transition path in Section 5.2, I find that λ^{total}, λ^{growth}, λ^{wealth}, λ^{s_C}, and $\lambda^{consumption}$ are 1.19, 1.09, 1.09, 1.05, and 1.04, respectively.[12] The welfare cost of rising wealth inequality is substantial, amounting to 19% of individual consumption in every period. Approximately half of this total cost originates from the general equilibrium effects on productivity slowdowns (λ^{growth}). The remainder originates from the wealth inequality, where λ^{wealth} is almost equally split between the limited average consumption (λ^{s_C}) and the increase in consumption inequality ($\lambda^{consumption}$).[13]

6 Conclusion

This paper studies the implications of rising wealth inequality on productivity slowdowns through the lens of an endogenous growth model with heterogeneous households. I emphasize a channel that connects rising inequality with productivity growth through aggregate

[12]See Appendix D.2 for details.

[13]For the empirical evidence on these trends, see Attanasio and Pistaferri (2014), Attanasio, Hurst and Pistaferri (2015), Fernald (2015), and Antolin-Diaz, Drechsel and Petrella (2017).

consumption demand. An increase in inequality decreases aggregate consumption demand because rich households save more. Given the limited consumption demand, firms have fewer incentives to spend on R&D because of smaller market sizes and lower profit gains. Thus, R&D-based productivity growth endogenously slows. Based on this channel, the model successfully replicates the macroeconomic trends in the US data since the 1980s, such as productivity slowdowns, low demand intensity, and low real interest rates. The welfare cost of rising wealth inequality is substantial because of an increase in consumption inequality, a decrease in demand intensity, and endogenous growth slowdowns.

A Proofs

Proof of Proposition 1. First, I focus on the Euler equation (8). When scaled consumption is given by $c_{i,t} = \bar{c}(x_{i,t}) = \zeta(x_{i,t} + \eta)^{\xi}$ for all t:

$$\frac{C_{i,t+1}}{C_{i,t}} = G_{t+1}\frac{c_{i,t+1}}{c_{i,t}} = G_{t+1}\left(\frac{x_{i,t+1}+\eta}{x_{i,t}+\eta}\right)^{\xi} = G_{t+1}\left(\frac{\frac{R_{t+1}}{G_{t+1}}(x_{i,t}-c_{i,t}) + m_{i,t+1} + \eta}{x_{i,t}+\eta}\right)^{\xi}$$

$$= G_{t+1}\left(\frac{\frac{R_{t+1}}{G_{t+1}}(x_{i,t}-\bar{c}(x_{i,t})) + \frac{R_{t+1}}{G_{t+1}}\bar{c}\left(\tilde{x}_{i,t}\right) + \left[\exp(z_{i,t+1}) - \frac{R_{t+1}}{G_{t+1}}\right](\tilde{x}_{i,t}+\eta) - \frac{G_{t+1}-R_{t+1}}{G_{t+1}}\eta + \eta}{x_{i,t}+\eta}\right)^{\xi}$$

because of budget constraint (6) and Assumption 1(EE5). Because $\tilde{x}_{i,0} = x_{i,0}$ by the initial condition (EE3) in Assumption 1, $\bar{c}(x_{i,0}) = \bar{c}(\tilde{x}_{i,0})$. Thus, $\frac{C_{i,1}}{C_{i,0}} = G_1\left(\frac{\exp(z_{i,1})(x_{i,0}+\eta)}{x_{i,0}+\eta}\right)^{\xi} = G_1\exp(\xi z_{i,1})$. Then, for $t = 0$, the Euler equation (8) simplifies to

$$1 \geq \mathbb{E}_0\left[\beta R_1 G_1^{-\gamma}\exp(-\xi\gamma z_{i,1})\right],$$

which holds with equality by Assumption 1(EE1-2). Thus, the proposed consumption function solves the Euler equation (8) with equality for $t = 0$. Furthermore, given $c_{i,0} = \bar{c}(x_{i,0})$, $x_{i,1} + \eta = \frac{R_1}{G_1}(x_{i,0} - c_{i,0}) + m_{i,0} + \eta = \exp(z_{i,1})(x_{i,0}+\eta) = \exp(z_{i,1})(\tilde{x}_{i,0}+\eta) = \tilde{x}_{i,1}+\eta$ by Assumption 1(EE4), implying that $\tilde{x}_{i,1} = x_{i,1}$. Therefore, by mathematical induction, the consumption function $c_{i,t} = \bar{c}(x_{i,t}) = \zeta(x_{i,t}+\eta)^{\xi}$ solves the Euler equation (8) with equality for all t. Furthermore, these consumption decisions imply that (1) $x_{i,t+1}+\eta = \exp(z_{i,t+1})(x_{i,t}+\eta)$ for all t, (2) $x_{i,t} = \tilde{x}_{i,t} > -\eta$ for all t, and (3) $c_{i,t+1} = \exp(\xi z_{i,t+1})c_{i,t}$ for all t (see also the proof of Proposition 2 for (1)-(3)).

For the transversality condition (9), it suffices to show that the condition holds under the balanced growth path to which the economy converges in the long run (Assumption 1 (TVC1)). Given the distributional family for $z_{i,t}$ (TVC2) and the random growth results for $c_{i,t}$ and $x_{i,t}$ in (1) and (3) above, Appendix C specifies γ_L and γ_U in (TVC3) such that $\max\{\gamma_L, 0\} < \gamma < \gamma_U$ is a sufficient condition for the transversality condition (9). Thus, Assumption 1 is a sufficient condition for the consumption function (10) to satisfy the Euler equation (8) and the transversality condition (9), which are sufficient conditions for the optimal consumption–savings decisions. □

Proof of Proposition 2. Suppose that $x_{i,t} = \tilde{x}_{i,t}$. Given $c_{i,t} = \bar{c}(c_{i,t})$:

$$\begin{aligned} x_{i,t+1} &= \frac{R_{t+1}}{G_{t+1}}(x_{i,t} - c_{i,t}) + m_{i,t+1} \\ &= \frac{R_{t+1}}{G_{t+1}}(x_{i,t} - \bar{c}(x_{i,t})) + \frac{R_{t+1}}{G_{t+1}}\bar{c}\left(\tilde{x}_{i,t}\right) + \left[\exp(z_{i,t+1}) - \frac{R_{t+1}}{G_{t+1}}\right](\tilde{x}_{i,t} + \eta) - \frac{G_{t+1} - R_{t+1}}{G_{t+1}}\eta \\ &= \exp(z_{i,t+1})(x_{i,t} + \eta) - \eta. \end{aligned}$$

Thus, $x_{i,t+1} + \eta = \exp(z_{i,t+1})(x_{i,t} + \eta) = \exp(z_{i,t+1})(\tilde{x}_{i,t} + \eta) = \tilde{x}_{i,t+1}$ (Assumption 1 (EE4)). Because $x_{i,0} = \tilde{x}_{i,0} > -\eta$ (EE3), by mathematical induction, I have:

$$\begin{gathered} x_{i,t} = \tilde{x}_{i,t} > -\eta, \\ x_{i,t+1} + \eta = \exp(z_{i,t+1})(x_{i,t} + \eta) \end{gathered}$$

for all t. Finally:

$$c_{i,t+1} = \bar{c}(x_{i,t+1}) = \zeta(x_{i,t+1} + \eta)^{\xi} = \exp(\xi z_{i,t+1})\zeta(x_{i,t} + \eta)^{\xi} = \exp(\xi z_{i,t+1})c_{i,t}$$

for all t. □

Proof of Proposition 3. Suppose $a_{i,t+1} \sim \mathcal{L}(\mu_{a,t+1}, \sigma^2_{a,t+1})$. To prove Proposition 3, it suffices to show that $\frac{1}{p_d}[f_{a,t+1} - (1 - p_d)f_{a+z',t+1}]$ is a well-defined pdf of a random variable $n_{i,t+1}$.

Fix $b \in \mathbb{R}$. Given Laplacian pdfs $f_{a,t}$ and $f_{z',t+1}$ (Assumption 2 (1-2)):

$$f_{a+z',t+1}(b) = \int f_{a,t}(b - s)f_{z',t+1}(s)ds$$

$$
\begin{aligned}
&= \int \frac{1}{\sqrt{2}\sigma_{a,t}} \exp\left(-\frac{|b-s-\mu_{a,t}|}{\sigma_{a,t}/\sqrt{2}}\right) \frac{1}{\sqrt{2}\sigma_{z,t+1}} \exp\left(-\frac{|s-\mu_{z,t+1}|}{\sigma_{z,t+1}/\sqrt{2}}\right) ds \\
&= \frac{1}{2\sigma_{a,t+1}\sigma_{z,t+1}} \int \exp\left(-\frac{|\hat{b}-\hat{s}|}{\sigma_{a,t+1}/\sqrt{2}} - \frac{|\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s} + \Delta\mu_{a,t+1}|}{\sigma_{z,t+1}/\sqrt{2}}\right) d\hat{s},
\end{aligned}
$$

where $\hat{s} \equiv \frac{\sigma_{a,t+1}}{\sigma_{a,t}}(s-\mu_{z,t+1}-\Delta\mu_{a,t+1})$, $\hat{b} \equiv \frac{\sigma_{a,t+1}}{\sigma_{a,t}}(b-\mu_{z,t+1}-\mu_{a,t+1})$, and $\Delta\mu_{a,t+1} \equiv \mu_{a,t+1}-\mu_{a,t}$.

For now, suppose that $\hat{b} \geq 0$. Because $-|\hat{b}-\hat{s}| \leq -\hat{b}+\hat{s} = -|\hat{b}|+\hat{s}$:

$$
\begin{aligned}
f_{a+z',t+1}(b) &\leq \frac{1}{2\sigma_{a,t+1}\sigma_{z,t+1}} \exp\left(-\frac{|\hat{b}|}{\sigma_{a,t+1}/\sqrt{2}}\right) \int \exp\left(\frac{\hat{s}}{\sigma_{a,t+1}/\sqrt{2}} - \frac{|\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s} + \Delta\mu_{a,t+1}|}{\sigma_{z,t+1}/\sqrt{2}}\right) d\hat{s} \\
&= f_{a,t+1}\left(\hat{b}+\mu_{a,t+1}\right) \left[\frac{1}{\sqrt{2}\sigma_{z,t+1}} \int \exp\left(\frac{\hat{s}}{\sigma_{a,t+1}/\sqrt{2}} - \frac{|\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s} + \Delta\mu_{a,t+1}|}{\sigma_{z,t+1}/\sqrt{2}}\right) d\hat{s}\right].
\end{aligned}
$$

Next, I show that with some algebra, the terms inside the square brackets reduce to $\exp\left(-\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}/\sqrt{2}}\right) \frac{\sigma_{a,t}\sigma_{a,t+1}}{\sigma_{a,t}^2-\sigma_{z,t+1}^2}$
Note that:

$$
\begin{aligned}
&\int \exp\left(\frac{\hat{s}}{\sigma_{a,t+1}/\sqrt{2}} - \frac{|\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s} + \Delta\mu_{a,t+1}|}{\sigma_{z,t+1}/\sqrt{2}}\right) d\hat{s} \\
= &\int_{-\infty}^{-\frac{\sigma_{a,t+1}}{\sigma_{a,t}}\Delta\mu_{a,t+1}} \exp\left(\frac{\hat{s}}{\sigma_{a,t+1}/\sqrt{2}} + \frac{\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s} + \Delta\mu_{a,t+1}}{\sigma_{z,t+1}/\sqrt{2}}\right) d\hat{s} \\
+ &\int_{-\frac{\sigma_{a,t+1}}{\sigma_{a,t}}\Delta\mu_{a,t+1}}^{\infty} \exp\left(\frac{\hat{s}}{\sigma_{a,t+1}/\sqrt{2}} - \frac{\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s} + \Delta\mu_{a,t+1}}{\sigma_{z,t+1}/\sqrt{2}}\right) d\hat{s} \\
= &\exp\left(\frac{\sqrt{2}\Delta\mu_{a,t+1}}{\sigma_{z,t+1}}\right) \frac{1}{\frac{\sqrt{2}}{\sigma_{a,t+1}} + \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}} \exp\left\{\left(\frac{\sqrt{2}}{\sigma_{a,t+1}} + \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\right)\hat{s}\right\}\Bigg|_{-\infty}^{-\frac{\sigma_{a,t+1}}{\sigma_{a,t}}\Delta\mu_{a,t+1}} \\
+ &\exp\left(-\frac{\sqrt{2}\Delta\mu_{a,t+1}}{\sigma_{z,t+1}}\right) \frac{1}{\frac{\sqrt{2}}{\sigma_{a,t+1}} - \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}} \exp\left\{\left(\frac{\sqrt{2}}{\sigma_{a,t+1}} - \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\right)\hat{s}\right\}\Bigg|_{-\frac{\sigma_{a,t+1}}{\sigma_{a,t}}\Delta\mu_{a,t+1}}^{\infty} \\
= &\exp\left(\frac{\sqrt{2}\Delta\mu_{a,t+1}}{\sigma_{z,t+1}}\right) \frac{1}{\frac{\sqrt{2}}{\sigma_{a,t+1}} + \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}} \exp\left\{-\sqrt{2}\left(1+\frac{\sigma_{a,t}}{\sigma_{z,t+1}}\right)\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}}\right\} \\
- &\exp\left(-\frac{\sqrt{2}\Delta\mu_{a,t+1}}{\sigma_{z,t+1}}\right) \frac{1}{\frac{\sqrt{2}}{\sigma_{a,t+1}} - \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}} \exp\left\{-\sqrt{2}\left(1-\frac{\sigma_{a,t}}{\sigma_{z,t+1}}\right)\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}}\right\} \\
= &\exp\left(-\sqrt{2}\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}}\right)\left(\frac{1}{\frac{\sqrt{2}}{\sigma_{a,t+1}} + \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}} - \frac{1}{\frac{\sqrt{2}}{\sigma_{a,t+1}} - \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}}}\right)
\end{aligned}
$$

$$= \exp\left(-\sqrt{2}\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}}\right)\frac{\sigma_{a,t+1}}{\sqrt{2}}\frac{2\sigma_{z,t+1}\sigma_{a,t}}{\sigma_{a,t}^2-\sigma_{z,t+1}^2},$$

where I used the assumption that $\sigma_{z,t+1} < \sigma_{a,t}$ (Assumption 2 (3)) and, therefore, $\frac{\sqrt{2}}{\sigma_{a,t+1}} - \frac{\sqrt{2}}{\sigma_{z,t+1}}\frac{\sigma_{a,t}}{\sigma_{a,t+1}} < 0$. Thus, I obtain that:

$$\begin{aligned}
f_{a+z',t+1}(b) &\le f_{a,t+1}\left(\hat{b}+\mu_{a,t+1}\right)\left[\frac{1}{\sqrt{2}\sigma_{z,t+1}}\int \exp\left(\frac{\hat{s}}{\sigma_{a,t+1}/\sqrt{2}} - \frac{\left|\frac{\sigma_{a,t}}{\sigma_{a,t+1}}\hat{s}+\Delta\mu_{a,t+1}\right|}{\sigma_{z,t+1}/\sqrt{2}}\right)d\hat{s}\right]\\
&= f_{a,t+1}\left(\hat{b}+\mu_{a,t+1}\right)\exp\left(-\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}/\sqrt{2}}\right)\frac{\sigma_{a,t}\sigma_{a,t+1}}{\sigma_{a,t}^2-\sigma_{z,t+1}^2}
\end{aligned}$$

as desired. When $\hat{b} < 0$, I use the fact that $-|\hat{b}-\hat{s}| \le \hat{b}-\hat{s} = -|\hat{b}|-\hat{s}$ and derive the same upper bound for $f_{a+z',t+1}(b)$.

I also compute an upper bound for $f_{a,t+1}\left(\hat{b}+\mu_{a,t+1}\right)$. By the reverse triangle inequality, $-|\hat{b}| = -|b-\mu_{a,t+1}-\mu_{z,t+1}| \le -|b-\mu_{a,t+1}| + |\mu_{z,t+1}|$. Thus:

$$\begin{aligned}
&f_{a,t+1}(\hat{b}+\mu_{a,t+1}) = \frac{1}{\sqrt{2}\sigma_{a,t+1}}\exp\left(-\frac{|\hat{b}|}{\sigma_{a,t+1}/\sqrt{2}}\right)\\
&= \frac{1}{\sqrt{2}\sigma_{a,t+1}}\exp\left(-\frac{\frac{\sigma_{a,t+1}}{\sigma_{a,t}}|b-\mu_{a,t+1}-\mu_{z,t+1}|}{\sigma_{a,t+1}/\sqrt{2}}\right)\\
&\le \frac{1}{\sqrt{2}\sigma_{a,t+1}}\exp\left(-\frac{|b-\mu_{a,t+1}|}{\sigma_{a,t+1}/\sqrt{2}}\right)\exp\left\{-\sqrt{2}|b-\mu_{a,t+1}|\left(\frac{1}{\sigma_{a,t}}-\frac{1}{\sigma_{a,t+1}}\right)\right\}\exp\left(\frac{|\mu_{z,t+1}|}{\sigma_{a,t}/\sqrt{2}}\right)\\
&\le f_{a,t+1}(b)\exp\left(\frac{|\mu_{z,t+1}|}{\sigma_{a,t}/\sqrt{2}}\right),
\end{aligned}$$

where I use the assumption that $\sigma_{a,t} \le \sigma_{a,t+1}$ in the last line (Assumption 2 (4)). Finally, I have:

$$\begin{aligned}
f_{a+z',t+1}(b) &\le f_{a,t+1}\left(\hat{b}+\mu_{a,t+1}\right)\exp\left(-\frac{\Delta\mu_{a,t+1}}{\sigma_{a,t}/\sqrt{2}}\right)\frac{\sigma_{a,t}\sigma_{a,t+1}}{\sigma_{a,t}^2-\sigma_{z,t+1}^2}\\
&\le f_{a,t+1}(b)\exp\left(\frac{|\mu_{z,t+1}|-\Delta\mu_{a,t+1}}{\sigma_{a,t}/\sqrt{2}}\right)\frac{\sigma_{a,t}\sigma_{a,t+1}}{\sigma_{a,t}^2-\sigma_{z,t+1}^2}\\
&= \mathcal{C}_{t+1}f_{a,t+1}(b) \quad \text{ for all } b\in\mathbb{R},
\end{aligned}$$

where $\mathcal{C}_{t+1} \equiv \frac{\sigma_{a,t}\sigma_{a,t+1}}{\sigma_{a,t}^2-\sigma_{z,t+1}^2}\exp\left(\frac{|\mu_{z,t+1}|-\mu_{a,t+1}+\mu_{a,t}}{\sigma_{a,t}/\sqrt{2}}\right)$. By integrating $f_{a+z',t+1}(b) \le \mathcal{C}_{t+1}f_{a,t+1}(b)$, I obtain $1 \le \mathcal{C}_{t+1}$.

Thus, if $p_d > 1 - \frac{1}{\mathcal{C}_{t+1}}$ (Assumption 2 (5)), then $(1-p_d)f_{a+z',t+1} < \frac{1}{\mathcal{C}_{t+1}} f_{a+z',t+1} \le f_{a,t+1}$; therefore, $f_{n,t+1} = p_d^{-1}[f_{a,t+1} - (1-p_d)f_{a+z',t+1}] > 0$. That is, $f_{n,t+1}$ is a well-defined pdf. $\square$

Proof of Proposition 4. Let $\Pi_{j,t}$ be the profit made by firm j in period t:

$$\Pi_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{1-\omega} Y_t - W_t \left(L_{Y,j,t} + L_{A,j,t} + L_M\right) - (r_t + \delta) K_{j,t}.$$

Firm j's Lagrangian is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{1}{\prod_{\tau=1}^{t}(1+r_\tau)} \Bigg\{ \Pi_{j,t} + \lambda_{mc,j,t}\left[K_{j,t}^{1-\alpha}\left(A_{j,t}L_{Y,j,t}\right)^{\alpha} - \left(\frac{P_{j,t}}{P_t}\right)^{-\omega} Y_t\right] + \lambda_{A,j,t}\left[\left(1+\theta L_{A,j,t}\right)A_{j,t} - A_{j,t+1}\right]\Bigg\}.$$

When firms are symmetric, the first-order conditions are as follows:

$$\begin{aligned}
P_{j,t} &: \lambda_{mc,t} = \mathcal{M}^{-1}, \\
L_{Y,j,t} &: W_t L_{Y,t} = \mathcal{M}^{-1}\alpha Y_t, \\
K_{j,t} &: r_t + \delta = \mathcal{M}^{-1}(1-\alpha)\frac{Y_t}{K_t}, \\
L_{A,j,t} &: \lambda_{A,t} = \frac{W_t}{\theta A_t}, \\
A_{j,t+1} &: 1 + r_{t+1} = \mathcal{M}^{-1}\alpha \frac{Y_{t+1}}{A_{t+1}}\frac{1}{\lambda_{A,t}} + \frac{\lambda_{A,t+1}}{\lambda_{A,t}}(1+\theta L_{A,t+1}).
\end{aligned}$$

I obtain Equation (14) from the first-order condition for $K_{j,t}$. Similarly, I derive Equation (15) by using the conditions for $L_{Y,j,t}$, $L_{A,j,t}$, and $A_{j,t+1}$. Equation (16) directly follows from Equations (14) and (15). $\square$

Proof of Proposition 5. I denote the left-hand side of Equation (16) as L. Then:

$$L(g, s_C) \equiv \frac{\mathcal{M}^{-1}(1-\alpha)}{1-s_C}\left[g + \delta + (1-\delta)s_C\right] = \mathcal{B}(s_C)\left(g + \mathcal{A}(s_C)\right),$$

where $\mathcal{A}(s_C) \equiv \delta + (1-\delta)s_C > 0$ and $\mathcal{B} \equiv \frac{\mathcal{M}^{-1}(1-\alpha)}{1-s_C} > 0$. For a fixed $s_C \in (0,1)$, L is a linear function in g with horizontal intercept $-\mathcal{A}$ and slope $\mathcal{B}$.

Similarly, the right-hand side, R, is given by:

$$\begin{aligned} R(g, s_C, s_A) &\equiv g + \theta \left(\frac{s_Y}{s_A^{\alpha} s_K^{1-\alpha}} \right)^{1/\alpha} \\ &= g + \frac{\theta}{s_A}(1 - s_C)^{\frac{\alpha-1}{\alpha}} G^{-1}(g + \mathcal{A}(s_C))^{1/\alpha} \\ &= g + \mathcal{C}(s_C) G^{-1}(g + \mathcal{A}(s_C))^{1/\alpha}, \end{aligned}$$

where $\mathcal{C}(s_C, s_A) \equiv \frac{\theta}{s_A}(1 - s_C)^{\frac{\alpha-1}{\alpha}} > 0$ for $s_A > 0$ and $G = 1 + g$. It follows that $R(-\mathcal{A}, s_C, s_A) = -\mathcal{A}$. Furthermore:

$$\begin{aligned} \frac{\partial R}{\partial g} &= 1 - \frac{\mathcal{C}(g + \mathcal{A})^{\frac{1}{\alpha}}}{G^2} + \frac{1}{\alpha} \frac{\mathcal{C}(g + \mathcal{A})^{\frac{1-\alpha}{\alpha}}}{G} \\ &\geq 1 - \frac{\mathcal{C}(g + \mathcal{A})^{\frac{1}{\alpha}}}{G^2} + \frac{1}{\alpha} \frac{\mathcal{C}(g + \mathcal{A})^{\frac{1}{\alpha}}}{G^2} \\ &= 1 + \frac{1 - \alpha}{\alpha} \frac{\mathcal{C}(g + \mathcal{A})^{\frac{1}{\alpha}}}{G^2}. \end{aligned}$$

In the second line, I use the fact that $0 < g + \mathcal{A} < G$ for all $g > -\mathcal{A}$ because $0 < \mathcal{A}(s_C) = 1 - (1 - \delta)(1 - s_C) \leq 1$. Therefore, $\frac{\partial R}{\partial g} > 1$ for all $g > -\mathcal{A}(s_C)$.

Fix $s_C \in (0, 1)$ and $s_A > 0$. I have $L(-\mathcal{A}, s_C) = 0 > -\mathcal{A} = R(-\mathcal{A}, s_C, s_A)$, $\frac{\partial L}{\partial g} = \mathcal{B}$, and $\frac{\partial R}{\partial g} > 1$ for all $g > -\mathcal{A}$. Therefore, if $\mathcal{B} < 1$, there exists a unique $g(s_C, s_A)$ such that $L(g(s_C, s_A), s_C) = R(g(s_C, s_A), s_C, s_A)$ and $g(s_C, s_A) > -\mathcal{A}(s_C)$. □

Proof of Proposition 6. I use the notation introduced in the proof of Proposition 5. I fix s_A to focus on the effects of s_C on $g(s_C, s_A)$. From Proposition 5, we know that $L(g(s_C, s_A), s_C) = R(g(s_C, s_A), s_C, s_A)$. Let $f(g, s_C, s_A)$ be $L - R$. Then, by the implicit function theorem, $\frac{\partial g(s_C, s_A)}{\partial s_C} = -\frac{\partial f}{\partial s_C} / \frac{\partial f}{\partial g}$, where $\frac{\partial f}{\partial g} = \mathcal{B} - \frac{\partial R}{\partial g} < 0$. Therefore,

$$\frac{\partial g(s_C, s_A)}{\partial s_C} > 0 \quad \text{if and only if} \quad \frac{\partial f}{\partial s_C} > 0.$$

With some algebra, I can show that:

$$\begin{aligned} \frac{\partial L(g, s_C)}{\partial s_C} &= \frac{1 - \alpha}{(1 - s_C)^2} \mathcal{M}^{-1} G, \quad \text{and} \\ \frac{\partial R(g, s_C, s_A)}{\partial s_C} &= \frac{1 - \alpha}{\alpha} \frac{R - g}{1 - s_C} + \frac{1 - \delta}{\alpha} \frac{R - g}{G - (1 - \delta)(1 - s_C)}. \end{aligned}$$

Furthermore,

$$\frac{\partial f}{\partial s_C} > 0 \iff \frac{\partial L}{\partial s_C} > \frac{\partial R}{\partial s_C} \iff \frac{(1-s_C)^2}{1-\alpha}\frac{\partial L}{\partial s_C} > \frac{(1-s_C)^2}{1-\alpha}\frac{\partial R}{\partial s_C},$$

where the last inequality simplifies to Equation (17) with χ being R.

Next, I turn to the special case in which $\alpha > s_C$, $\alpha > \frac{1}{2}$, $g(s_C, s_A) > -\left[1 + \frac{(1-s_C)\mathcal{M}}{2\alpha-1}\right]^{-1}$, and δ is sufficiently large. I first show that $g(s_C, s_A)$ and $\chi = R(g(s_C, s_A), s_C, s_A)$ are bounded as δ approaches 1. When $\delta \to 1$, $\mathcal{A} \to 1$. Thus, $L(g, s_C) \to \mathcal{B}(s_C)(1+g)$, where $0 < \mathcal{B} < 1$ by assumption. Similarly, $R(g, s_C, s_A)$ converges to $g + \mathcal{C}(s_C)(1+g)^{\frac{1-\alpha}{\alpha}}$. Therefore, in the limit, the graph of L starts from $(-1, 0)$, and its slope is $0 < \mathcal{B} < 1$. On the other hand, the graph of R starts from $(-1, -1)$, and its slope is greater than 1. As a result, as $\delta \to 1$, $g(s_C)$ is bounded between -1 and $\frac{\mathcal{B}}{1-\mathcal{B}}$, where $\mathcal{B}(1+g)$ and g coincide when $g = \frac{\mathcal{B}}{1-\mathcal{B}}$. Furthermore, $\chi = R(g(s_C, s_A), s_C, s_A) = L(g(s_C, s_A), s_C)$ is bounded because its limit $\mathcal{B}(s_C)(1+g(s_C, s_A))$ is bounded.

Since χ and $g(s_C, s_A)$ are bounded, the right-hand side of Equation (17) converges to $\frac{1-s_C}{\alpha}(\chi - g(s_C, s_A))$ as $\delta \to 1$. Because $\chi = R(g(s_C, s_A), s_C, s_A) = L(g(s_C, s_A), s_C)$, Equation (17) reduces to $\mathcal{M}^{-1}(1 + g(s_C, s_A)) > \frac{1-s_C}{\alpha}(\mathcal{B}(s_C)(1 + g(s_C, s_A)) - g(s_C, s_A))$. This last inequality is equivalent to $g(s_C, s_A) > -\left[1 + \frac{(1-s_C)\mathcal{M}}{2\alpha-1}\right]^{-1}$ when $\alpha > \frac{1}{2}$. □

Proof of Proposition 7. For a large x_i such that $\log(x_i + \eta) \geq \mu_a$:

$$\begin{aligned} Pr(x_i \geq \underline{x}) = Pr(a_i \geq \log(\underline{x} + \eta)) &= \int_{\log(\underline{x}+\eta)}^{\infty} \frac{1}{\sqrt{2}\sigma_a} \exp\left(-\frac{b-\mu_a}{\sigma_a/\sqrt{2}}\right) db \\ &= \frac{1}{2}\exp\left(\frac{\mu_a}{\sigma_a/\sqrt{2}}\right)(\underline{x}+\eta)^{-\frac{\sqrt{2}}{\sigma_a}} \propto \underline{x}^{-\frac{\sqrt{2}}{\sigma_a}}. \end{aligned}$$

Thus, the right tail of x_i has a Pareto distribution with Pareto coefficient $\frac{\sqrt{2}}{\sigma_a}$.

For aggregate wealth and consumption, I use the facts that $\mathbb{E}\{\exp[\mathcal{L}(\mu, \sigma^2)]\} = \frac{\exp(\mu)}{1-\frac{1}{2}\sigma^2}$ (see Kotz, Kozubowski and Podgórski, 2001) and $\log c_i = \log\zeta + \xi \log a_i \sim \mathcal{L}(\log\zeta + \xi\mu_a, \xi^2\sigma_a^2)$.

For the comparative statics for s_C, note that $x = \hat{x} - \eta$, where $\hat{x} \equiv \frac{\exp(\mu_a)}{1-0.5\sigma_a^2}$. It follows that $dx = d\hat{x} - d\eta = \hat{x}d\mu_a + \frac{0.5\hat{x}}{1-0.5\sigma_a^2}d\sigma_a^2 - d\eta = 0$. By assumption, $d\eta = 0$. Thus, $\frac{d\mu_a}{d\sigma_a^2}|_x = -\frac{0.5}{1-0.5\sigma_a^2}$.

The total differential of $s_C = \frac{c}{x}$ with respect to μ_a and σ_a^2 yields:

$$ds_C = \frac{dc}{x} - \frac{c}{x^2}dx = \frac{1}{x}\left(\xi c d\mu_a + \frac{0.5\xi^2 c}{1-0.5\xi^2\sigma_a^2}d\sigma_a^2\right) - \frac{c}{x^2}dx.$$

Finally:

$$\frac{ds_C}{d\sigma_a^2}|_x = \frac{1}{x}\left(\xi c \frac{d\mu_a}{d\sigma_a^2}|_x + \frac{0.5\xi^2 c}{1-0.5\xi^2\sigma_a^2}\right) = -\frac{0.5\xi c}{x}\left(\frac{1}{1-0.5\sigma_a^2} - \frac{\xi}{1-0.5\xi^2\sigma_a^2}\right)$$
$$= -\frac{1}{2}\frac{1+0.5\xi\sigma_a^2}{(1-0.5\xi^2\sigma_a^2)(1-0.5\sigma_a^2)}\xi(1-\xi)s_C = -\mathcal{D}s_C,$$

where $\mathcal{D} = \frac{1}{2}\frac{1+0.5\xi\sigma_a^2}{(1-0.5\xi^2\sigma_a^2)(1-0.5\sigma_a^2)}\xi(1-\xi) > 0$. It is clear that $\frac{ds_C}{d\sigma_a^2}|_x < 0$. □

B Data appendix

B.1 Cross-country data

This appendix lists the data sources and details on the construction of the time series presented in Section 2. Following Piketty and Zucman (2014), the sample of rich countries in this paper includes the G7 countries (Canada, France, Germany, Italy, Japan, the UK,and the US) and Australia.

(a) **Top 10% wealth shares.** Annual series are obtained from the World Inequality Database. As of September 2021, this series is available for the US, the UK, and France out of the eight countries.

(b) **Top 10% income shares.** Annual data are obtained from the World Inequality Database.

(c) **RGDP per capita growth rates.** I use annual real GDP per capita data from the World Bank (constant local currency units, NY.GDP.PCAP.KN). Canadian real GDP data are available from 1997. I take 9-year moving averages of the growth rates to concentrate on trend-level variation.

(d) **Consumption-to-wealth ratios.** For private consumption, I use final consumption expenditure data for households and nonprofit institutions serving households (NPISHs) from the World Bank (current local currency units, NE.CON.PRVT.CN). The denominator is net private wealth measured in current local currency units. The data source is the World Inequality Database (see also Piketty and Zucman, 2014). The consumption and wealth data are annual.

(e) **R&D intensity.** Following Benigno and Fornaro (2018), I define R&D intensity as the ratio of business R&D investment to the R&D stock $RD_{inv,t}/RD_{stock,t}$ in each year t. By using real R&D investment data, the R&D stock in each country is computed as follows:

$$RD_{stock,t+1} = RD_{inv,t} + (1 - \delta_{RD})RD_{stock,t},$$

given the initial condition $RD_{stock,0} = RD_{inv,0}/\delta_{RD}$, where δ_{RD} represents the depreciation rate of the R&D stock. I fix δ_{RD} at the traditionally assumed value of 0.15. The results are robust to assuming an alternative initial condition $RD_{stock,0} = 0$.

The business R&D investment data in current local currencies and the GDP deflators are obtained from the OECD database. Using the real R&D investment series, the R&D stock and corresponding intensity measures are constructed for each country from 1970 to 2018. The missing values of $RD_{inv,t}$ are interpolated from the adjacent observations. To focus on slow-moving components, I take 9-year moving averages of the R&D intensity series for each country.

B.2 Wealth, income, and consumption in the PSID

This appendix covers the details of the variables constructed from the Panel Study of Income Dynamics (PSID) data, which are analyzed in Section 5.

(a) **Sample periods.** The PSID wealth data are available in the 1984, 1989, 1994, and 1999 waves and every two years after that.

(b) **PSID waves and calendar years.** I consider variables in the PSID 1984 wave to be observations from the calendar year of 1983. The same timing convention is used for the other waves.

(c) **Data cleaning.** I use the number of family members to construct the per capita net wealth and income variables. Additionally, I drop observations if the head of the family unit is younger than 20 or older than 65.

(d) **Definitions of variables.**

- **Wealth.** Net wealth is defined as the sum of the values of a farm or business, checking or savings accounts, money market funds, certificates of deposit, government bonds or treasury bills, real estate, shares of stock in publicly held corporations, stock mutual funds or investment trusts, private annuities or IRAs, other assets, and the net value of any cars, trucks, motor homes, trailer, or boat less the sum of liabilities from a farm, a business, or real estate, credit card or store card debt, student loan debt, medical bills, legal bills, loans from relatives, and other debt.
- **Income.** Income includes taxable income, transfers, and social security income.
- **Wealth inclusive of income.** Wealth inclusive of income is the sum of net wealth and total income.
- **Consumption.** As in Attanasio and Pistaferri (2014), consumption includes expenditures on food, rent, home insurance, utilities, car insurance, car repairs, transportation, school, child care, health insurance, and out-of-pocket medical expenses.

B.3 Macroeconomic trends in the US economy

This appendix presents the details of the empirical time series shown in Figure 5.

(a) **Consumption-to-wealth ratios.** Following Laibson (1999), aggregate consumption includes personal consumption expenditure and government consumption. Aggregate wealth inclusive of income is the sum of net national wealth K_t and net national income $Y_t - \delta K_t$. I obtain consumption data from the National Income and Product Accounts and wealth and national income series from Tables B.1 and S.1 in the Financial Accounts of the United States - Z.1.

(b) **Growth rates.** To focus on the slow-moving elements of economic growth, I plot the moving averages of the real GDP per capita growth rate with a window size of 9 years.

(c) **Real interest rates.** I plot the natural rate of interest estimated by Laubach and Williams (2003, 2016) using the right vertical axis.

(d) **Capital-to-income ratios.** I use the time series computed by Piketty and Zucman (2014) for the US economy.

C Transversality conditions and the relative risk aversion coefficient

In this appendix, I present the details on the calculation of γ_L and γ_U in Assumption 1(TVC3). Note that the transversality condition is given by:

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\beta^T \left(C_{i,T}\right)^{-\gamma} X_{i,T}\right] = 0.$$

This condition holds if:

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\beta^T \left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma} \frac{X_{i,T}+\eta_T}{X_{i,0}+\eta}\right] = 0 \quad \text{and} \quad \lim_{T\to\infty} \mathbb{E}_0 \left[\beta^T \left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma} \frac{\eta_T}{\eta}\right] = 0$$

because these two equations imply that $\lim_{T\to\infty} \mathbb{E}_0 \left[\beta^T \left(C_{i,T}\right)^{-\gamma} \left(X_{i,T}+\eta_T\right)\right] = 0$ and $\lim_{T\to\infty}$ $\mathbb{E}_0[\beta^T (C_{i,T})^{-\gamma}\eta_T] = 0$. Thus, I derive a sufficient condition for these two conditions.

Using the random growth results in Proposition 2, I have $C_{i,t} = G_t \exp(\xi z_{i,t}) C_{i,t-1}$ and $X_{i,t} + \eta_t = G_t \exp(z_{i,t})(X_{i,t-1} + \eta_{t-1})$. Because $\eta_0 = \eta$. I obtain:

$$\begin{aligned}
\beta^T \left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma} \frac{X_{i,T}+\eta_T}{X_{i,0}+\eta} &= \beta^T \left[\prod_{\tau=1}^{T} G_\tau^{-\gamma} \exp(-\xi\gamma z_{i,\tau})\right] \left[\prod_{\tau=1}^{T} G_\tau \exp(z_{i,\tau})\right] \\
&= \beta^T \left[\prod_{\tau=1}^{T} G_\tau^{1-\gamma} \exp((1-\xi\gamma) z_{i,\tau})\right].
\end{aligned}$$

Let $\mathcal{E}_t(\nu)$ be $\mathbb{E}[\exp(\nu z_{i,t})]$. Then,

$$\mathbb{E}_0 \left[\beta^T \left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma} \frac{X_{i,T}+\eta_T}{X_{i,0}+\eta}\right] = \prod_{\tau=1}^{T} \beta G_\tau^{1-\gamma} \mathcal{E}_\tau(1-\xi\gamma).$$

Because g_t, $\mu_{z,t}$, and $\sigma_{z,t}$ converge to g, μ_z, and σ_z, respectively, as $t \to \infty$. $\mathcal{E}_t(1-\xi\gamma)$ converges to $\mathcal{E}(1-\xi\gamma)$ as $t \to \infty$. By continuity and given that $\beta G^{1-\gamma}\mathcal{E}(1-\xi\gamma) > 0$, it is clear that $\lim_{T\to\infty} \mathbb{E}_0 \left[\beta^T \left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma} \frac{X_{i,T}+\eta_T}{X_{i,0}+\eta}\right] = 0$ if $\beta G^{1-\gamma}\mathcal{E}(1-\xi\gamma) < 1$, or, equivalently,

$$\mathcal{TV}_x(\gamma) \equiv -\rho + (1-\gamma)\hat{g} + \log(\mathcal{E}(1-\xi\gamma)) < 0, \tag{C.1}$$

where $\hat{g} \equiv \log(G)$.

Similarly:

$$\mathbb{E}_0\left[\beta^T\left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma}\frac{\eta_T}{\eta}\right] = \mathbb{E}_0\left\{\beta^T\left[\prod_{\tau=1}^{T} G_\tau^{-\gamma}\exp(-\xi\gamma z_{i,\tau})\right]\prod_{\tau=1}^{T} G_\tau\right\}$$
$$= \prod_{\tau=1}^{T} \beta G_\tau^{1-\gamma}\mathcal{E}_\tau(-\xi\gamma).$$

Thus, $\lim_{T\to\infty}\mathbb{E}_0\left[\beta^T\left(\frac{C_{i,T}}{C_{i,0}}\right)^{-\gamma}\frac{\eta_T}{\eta}\right] = 0$ if $\beta G^{1-\gamma}\mathcal{E}(-\xi\gamma) < 1$, or, equivalently:

$$\mathcal{TV}_\eta(\gamma) \equiv -\rho + (1-\gamma)\hat{g} + \log(\mathcal{E}(-\xi\gamma)) < 0. \tag{C.2}$$

When $z_{i,t} \sim \mathcal{L}(\mu_{z,t}, \sigma_{z,t}^2)$:

$$\mathcal{E}(\nu) = \frac{\exp(\nu\mu_z)}{1-\frac{1}{2}\nu^2\sigma_z^2} \quad \text{for} \quad |\nu| < \frac{\sqrt{2}}{\sigma_z}.$$

Given that $|1-\xi\gamma| < \frac{\sqrt{2}}{\sigma_z}$:

$$\mathcal{TV}_x(\gamma) = -\rho + (1-\gamma)\hat{g} + (1-\xi\gamma)\mu_z - \log\left(1 - \frac{1}{2}(1-\xi\gamma)^2\sigma_z^2\right).$$

It is straightforward to show that $\mathcal{TV}_x''(\gamma) > 0$; therefore, $\mathcal{TV}_x$ is strictly convex. Furthermore, as $1-\xi\gamma$ approaches $\pm\frac{\sqrt{2}}{\sigma_z}$, $\mathcal{TV}_x$ diverges to ∞. As a result, given that $|1-\xi\gamma| < \frac{\sqrt{2}}{\sigma_z}$, Equation (C.1) is equivalent to:

$$\gamma_{x,L} < \gamma < \gamma_{x,U},$$

where $\gamma_{x,L}$ and $\gamma_{x,U}$ are two distinct real roots of $\mathcal{TV}_x(\gamma) = 0$ such that $|1-\xi\gamma| < \frac{\sqrt{2}}{\sigma_z}$. If a real root of this equation does not exist, I define $\gamma_{x,L} = \infty$ and $\gamma_{x,U} = -\infty$. Similarly, given that $|\xi\gamma| < \frac{\sqrt{2}}{\sigma_z}$:

$$\mathcal{TV}_\eta(\gamma) = -\rho + (1-\gamma)\hat{g} + (-\xi\gamma)\mu_z - \log\left(1 - \frac{1}{2}(-\xi\gamma)^2\sigma_z^2\right),$$

which is strictly convex and diverges to ∞ as γ approaches $\pm\xi^{-1}\frac{\sqrt{2}}{\sigma_z}$. I define $\gamma_{\eta,L}$ and $\gamma_{\eta,U}$ accordingly.

Thus, if $\max\{\gamma_L, 0\} < \gamma < \gamma_U$, the transversality condition is satisfied, where

$$\gamma_L = \max\{\gamma_{x,L}, \gamma_{\eta,L}\} \quad \text{and} \quad \gamma_U = \min\{\gamma_{x,U}, \gamma_{\eta,U}\}.$$

Finally, in the special case where $\eta = 0$, $\eta_t = 0$ for all t, $\mathcal{TV}_\eta(\gamma) < 0$ is redundant. Therefore, in this case, $\gamma_L = \gamma_{x,L}$ and $\gamma_U = \gamma_{x,U}$.

D Solution methods for the quantitative analysis

This section covers the computational methods used for the quantitative analysis in Sections 5.2 and 5.3.

D.1 Transition dynamics

I first express the variables that grow in terms relative to X_t, e.g., $s_{Y,t} = \frac{Y_t}{X_t}$, $s_{W,t} = \frac{W_t}{X_t}$, $s_{K,t} = \frac{K_t}{X_t}$, and $s_{A,t} = \frac{A_t}{X_t}$. Because I consider a growth path such that X_t and A_t grow at the rate of g_t, $s_{A,t} = s_A$.

From the firms' first-order conditions in the proof of Proposition 4, I obtain the following equilibrium conditions:

$$s_{W,t} L_{Y,t} = \mathcal{M}^{-1} \alpha s_{Y,t}, \tag{D.1}$$

$$r_t + \delta = \mathcal{M}^{-1}(1-\alpha)\frac{s_{Y,t}}{s_{K,t}}, \tag{D.2}$$

$$1 + r_{t+1} = \frac{s_{W,t+1}}{s_{W,t}}(1 + g_{t+2} + \theta L_{Y,t+1}). \tag{D.3}$$

Furthermore, the production function (2), the aggregate resource constraint ($X_t = C_t + K_{t+1}$), and the zero-profit condition preventing new entry yield the following equilibrium conditions:

$$s_{Y,t} = s_{K,t}^{1-\alpha} s_A^{\alpha} L_{Y,t}^{\alpha}, \tag{D.4}$$

$$s_{K,t+1} = \frac{1 - s_{C,t}}{1 + g_{t+1}}, \tag{D.5}$$

$$0 = s_{Y,t} - s_{W,t}(L_{Y,t} + g_{t+1}/\theta + L_M) - (r_t + \delta)s_{K,t}, \tag{D.6}$$

where I use the fact that $g_t = \theta L_{A,t}$ in the last equation.

The economy is assumed to be on a balanced growth path in 1983. Thus, K_{1984} and $g_{1984} = \theta L_{A,1983}$ are determined accordingly. Because $X_{1984} = (1 + g_{1984})X_{1983}$, $s_{K,1984}$ constitutes an initial condition for the transition dynamics that start in 1984. I assume that the economy converges to its new balanced growth path equilibrium with a high level of wealth inequality within 50 years after the wealth distribution stabilizes in 2019. I use the following algorithm to derive this transition path.

1. Using the assumed path of $\mu_{a,t}$ and $\sigma_{a,t}$, calculate $\{s_{C,t}\}$.

2. Guess $s_{Y,1984}$.

3. Compute $L_{Y,1984}$ from Equation (D.4), $s_{W,1984}$ from Equation (D.1), r_{1984} from Equation (D.2), g_{1985} from Equation (D.6), and $s_{K,1985}$ from Equation (D.5).

4. Guess $s_{Y,1985}$.

5. Compute $L_{Y,1985}$ from Equation (D.4), $s_{W,1985}$ from Equation (D.1), r_{1985} from Equation (D.2), g_{1986} from Equation (D.6), and $s_{K,1986}$ from Equation (D.5).

6. Calculate another r_{1985} from Equation (D.3) using the values of $s_{W,1985}$, g_{1986}, and $L_{Y,1985}$ obtained from Step 5 and that of $s_{W,1984}$ from Step 3. If this r_{1985} is different from the r_{1985} computed in Step 5, go to step 4.

7. Iterate forward.

8. If g_{2069} is not equal to the growth rate in the posttransition balanced growth path equilibrium, go to step 2.

After deriving the dynamics of the aggregate variables, I calculate $\mu_{z,t+1}$ and $\sigma_{z,t+1}$ from the Euler equation ($\mathbb{E}[\beta R_{t+1} G_{t+1}^{-\gamma} \exp(-\xi\gamma z_{i,t+1})] = 1$) and the labor market clearing condition ($\mathbb{E}[\exp(z_{i,t+1})] = 1$).[14] Finally, $f_{n,t+1}$ follows from Equation (12).

[14] I assume that $\mu_{z,1984}$ and $\sigma_{z,1984}$ are equal to their 1983 values. Note that the Euler equation evaluated in 1983, $1 = \beta\mathbb{E}_{1983}[R_{t+1} G_{t+1}^{-\gamma} \exp(-\xi\gamma z_{i,1984})]$, holds under perfect foresight in 1983, i.e., foresight of the balanced growth path equilibrium. However, a structural change occured in 1984, and R_{1984} deviated from the previously expected value. Thus, this Euler equation with the realized value of R_{1984} is not used to calculate the corresponding values of $\mu_{z,1984}$ and $\sigma_{z,1984}$.

D.2 Welfare analysis

This section presents the computational details for the social welfare function in Equation (19) and for λ^{total}, λ^{growth}, and $\lambda^{dispersion}$. Note that $(\lambda C_{i,t})^{1-\gamma} = \left(\prod_{\tau=1}^{t} G_\tau\right)^{1-\gamma} (\lambda c_{i,t})^{1-\gamma}$. Then:

$$\int \frac{(\lambda C_{i,t})^{1-\gamma} - 1}{1-\gamma}\, di = \lambda^{1-\gamma} \left(\prod_{\tau=1}^{t} G_\tau\right)^{1-\gamma} \int \frac{c_{i,t}^{1-\gamma}}{1-\gamma}\, di - \frac{1}{1-\gamma}.$$

I denote $\int \frac{c_{i,t}^{1-\gamma}}{1-\gamma}\, di$ by v_t. Because $c_{i,t}^{1-\gamma} = \zeta^{1-\gamma} \exp[\xi(1-\gamma)a_{i,t}]$ and $a_{i,t} \sim \mathcal{L}(\mu_{a,t}, \sigma_{a,t}^2)$, I obtain:

$$v_t = v(\mu_{a,t}, \sigma_{a,t}) = \frac{\zeta^{1-\gamma} \exp[\xi(1-\gamma)\mu_{a,t}]}{1 - 0.5\xi^2(1-\gamma)^2\sigma_{a,t}^2} \frac{1}{1-\gamma}$$

if $|\xi(1-\gamma)\sigma_{a,t}| < \sqrt{2}$. This inequality condition holds for all quantitative exercises in this paper. Thus:

$$SW\left(\lambda, \{g_t\}, \{\mu_{a,t}, \sigma_{a,t}\}\right) = \lambda^{1-\gamma} \sum_t \left\{\beta^t \left(\prod_{\tau=1}^{t} G_\tau\right)^{1-\gamma} v(\mu_{a,t}, \sigma_{a,t})\right\} - \frac{1}{(1-\gamma)(1-\beta)}.$$

I define λ^{total}, λ^{growth}, and λ^{wealth} as follows:

$$\begin{aligned} SW(\lambda = 1, g_{1983}, \mu_{a,1983}, \sigma_{a,1983}) &= SW(\lambda^{growth}, \{g_t\}, \mu_{a,1983}, \sigma_{a,1983}) \\ &= SW(\lambda^{growth}\lambda^{wealth}, \{g_t\}, \{\mu_{a,t}, \sigma_{a,t}\}), \text{ and} \\ \lambda^{total} &= \lambda^{growth}\lambda^{wealth}. \end{aligned}$$

The social welfare functions are evaluated in 1984 when the transition begins. With some algebra, it follows that:

$$\begin{aligned} \lambda^{growth} &= \left\{\frac{v_{1983}/(1-\beta G_{1983}^{1-\gamma})}{\sum_{t=0}^{\infty}\left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} v_{1983}\right]}\right\}^{\frac{1}{1-\gamma}}, \text{ and} \\ \lambda^{wealth} &= \left\{\frac{\sum_{t=0}^{\infty}\left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} v_{1983}\right]}{\sum_{t=0}^{\infty}\left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} v_{t+1984}\right]}\right\}^{\frac{1}{1-\gamma}}. \end{aligned}$$

For the decomposition of λ^{wealth} into λ^{s_C} and $\lambda^{consumption}$, I note the following fact:

$$s_{C,t} = \frac{c_t}{x_t} = \frac{\zeta \exp(\xi \mu_{a,t})}{1 - 0.5\xi^2 \sigma_{a,t}^2},$$
$$\log(c_{i,t}/c_t) \sim \mathcal{L}(\log(1 - 0.5\xi^2\sigma_{a,t}^2), \xi^2\sigma_{a,t}^2).$$

In the first line, I use the fact that scaled aggregate wealth x_t equals one. Because $v_t = \int \frac{c_{i,t}^{1-\gamma}}{1-\gamma}\, di = c_t^{1-\gamma} \int \frac{(c_{i,t}/c_t)^{1-\gamma}}{1-\gamma}\, di$, I write:

$$v_t = s_{C,t}^{1-\gamma} \hat{v}_t,$$

where $\hat{v}_t = \hat{v}(\sigma_{a,t}) = \frac{(1-0.5\xi^2\sigma_{a,t}^2)^{1-\gamma}}{1-0.5\xi^2(1-\gamma)^2\sigma_{a,t}^2} \frac{1}{1-\gamma}$. Then, λ^{s_C} and $\lambda^{consumption}$ are defined as follows:

$$\lambda^{s_C} = \left\{ \frac{\sum_{t=0}^{\infty} \left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} v_{1983}\right]}{\sum_{t=0}^{\infty} \left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} s_{C,t+1984}^{1-\gamma}\hat{v}_{1983}\right]} \right\}^{\frac{1}{1-\gamma}}, \text{ and}$$
$$\lambda^{consumption} = \left\{ \frac{\sum_{t=0}^{\infty} \left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} s_{C,t+1984}^{1-\gamma}\hat{v}_{1983}\right]}{\sum_{t=0}^{\infty} \left[\beta^t \left(\prod_{\tau=1}^{t} G_{\tau+1984}\right)^{1-\gamma} v_{t+1984}\right]} \right\}^{\frac{1}{1-\gamma}},$$

where $v_{t+1984} = s_{C,t+1984}^{1-\gamma}\hat{v}_{t+1983}$ and $\lambda^{wealth} = \lambda^{s_C}\lambda^{consumption}$.

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